Alternatives for Locomotion Control

Chapter 6 presented by Peyman Mohajerian
Alternative to control of algorithm by changing some of the design and implementation decisions, e.g.:

- Three-part decomposition
- Use of foot placement and symmetry
- Virtual Legs
Instead of fixed thrust, adjust the energy injected on each hop.

Advantages:
\- Less susceptible to frictional variations
\- Correct for sweeping of the leg

Disadvantage:
\- More complicated control
Fixed Hopping Height- Schematic

Leg:
- Sliding joint
- Spring
- Actuator
- Mechanical stop

Actuator and spring act in series to lengthen and shorten the leg
Positive Work:
Support: lengthening actuator
Flight: shortening actuator

Negative Work:
Support: shorting actuator
Flight: lengthening actuator
During leg support, spring-mass oscillator:

\[ \omega_n = \sqrt{\frac{k_\ell}{m}}; \quad T_s = \frac{\pi}{\omega_n} = \pi \sqrt{\frac{m}{k_\ell}}. \]

Parabolic trajectory during flight:

\[ T_f = \frac{2z}{g} = \sqrt{\frac{8H}{g}}. \]

Period of a full hoping cycle:

\[ T = \pi \sqrt{\frac{m}{k_\ell}} + \sqrt{\frac{8H}{g}}. \]
Fixed Hopping Height- Energy

The total vertical energy at any time:

\[ E = m_\ell g z_1 + mg z_2 + \frac{1}{2} m_\ell \dot{z}_1^2 + \frac{1}{2} m \dot{z}_2^2 + \frac{1}{2} k_\ell (r_{s0} - r + w_\ell)^2. \]

Touchdown energy lost:

\[ \Delta E_{td} = \frac{1}{2} m_\ell \dot{z}_{1,td-}^2, \]

Equating linear moment:

\[ m \dot{z}_{2,io-} = (m + m_\ell) \dot{z}_{2,io+}, \]

\[ \dot{z}_{2,io-} = \frac{m}{m_\ell + m} \dot{z}_{2,io-}. \]
Fixed Hopping Height- Energy

Energy lost during liftoff:

$$\Delta E_{lo} = -\frac{m_\ell m}{2(m_\ell + m)} \dot{z}^2_{2,lo}.$$  

During stance total hopping energy for next flight:

$$E_f = \frac{m}{m_\ell + m} \left( m_\ell g z_1 + m g z_2 + \frac{1}{2} m_\ell \dot{z}^2_1 + \frac{1}{2} m \dot{z}^2_2 + \frac{1}{2} (k_\ell r^2_{s\Delta}) + \frac{1}{2} k_g \dot{z}^2_0 \right).$$
Fixed Hopping Height- Energy

Total vertical energy to hop to $H$:

$$E_H = m_v g (H + l_1) + m_v g (H + r_{30} + l_2).$$

The leg actuation must be:

$$\Delta w = -r_{3\Delta} + \sqrt{r_{3}^{2} + \frac{2 \Delta E}{k_v}}.$$
Fixed Hopping Height - Simulation
Hopping Strategies

1- Leg shortening at lift-off
   \  Ground clearance- uneven train
   \  Reduces leg’s moment of inertia

2- Leg shortening at top
   \  Slower actuator, increase time between vertical actuation

3- Leg shortening at touchdown
   \  Ground impact force to foot reduced- period of acceleration to ground speed increase.

   Human apply items 1 and 3 at the expense of extra lengthening and shorting!!!
Alternative Three-part Control

So far assignment of control action to variable of control:

- Forward foot placement - forward velocity
- Hip torque - body attitude
- Leg thrust - hopping height

Alternative, leg sweeping algorithm:

- Forward foot placement - body attitude
- Hip torque - forward velocity
To keep hopping at a desired constant speed, no net horizontal force.

Using hip torque sweep leg and foot backward during stance for a specific speed.
Leg Sweeping Algorithm

Hip servo:

\[ \tau = -k_p(\gamma - \gamma_d) - k_v(\dot{\gamma}), \]

Forward position of center of mass:

\[ x_{cg} = \frac{(l_1 - r)m_\ell \sin(\theta) + l_2 m \sin(\phi)}{m_\ell + m}. \]

Neutral point:

\[ x_{f0} = \frac{i T_3}{2}. \]

To accelerate attitude, foot displacement:

\[ x_{f\Delta} = k_\phi(\phi - \phi_d) + k_\dot{\phi} \dot{\phi}, \]
At touchdown, forward position:

\[ x_f = x_{cg} + x_f^0 + x_f \Delta. \]

During stand, foot should move backward with respect to the center of mass, horizontal trajectory:

\[ x_f(t) = x_{cg} + x_f^0 + x_f \Delta - \dot{x}_d(t - t_{td}), \]

The kinematics of the leg:

\[ x_f = -r \sin \theta, \]

Eliminate dependence on leg angle:

\[ -r \sin \theta = \frac{(l_1 - r)m_f \sin(\theta) + l_2 m \sin(\phi)}{m_f + m} + x_f^0 + x_f \Delta - \dot{x}_d(t - t_{td}). \quad (6.22) \]
Leg, body, and hip angle are related: \[ \gamma = \phi - \theta. \]

Based on forward speed of body, duration of stance, and geometry of the system, hip angle is:

\[ \gamma_d = \phi + \arcsin \left( \frac{l_2 m \sin(\phi) + (m_{\ell} + m)(x f_0 + x f_{\Delta} - \dot{z}_d(t - t_{ld}))}{l_1 m_{\ell} + rm} \right). \]
Quadruped Running - Coordinates

Machine Coordinate for quadruped:
- Preferred direction of travel
- Asymmetry in forward and lateral behavior

World coordinates for hopping:
- No orientation preference
Tight coupling of leg sequences.

Alternatives:

\[ \text{Treat each leg independently} \]
\[ \text{Legs coupled for vertical thrust only} \]
\[ \text{Velocity control based on independent leg but position based on the coupled overall velocity of the body} \]
Quadruped Running - Coordinating Leg Thrust

Simultaneous thrust delivery to all legs on ground - avoid tipping.

- Servo leg length during flight
- Servo leg length during support - same axial force
- Servo leg length during support - keep body level
- Thrust independently according to separate state of each leg similar to single leg hoping
Velocity Control Algorithm:
- Sweep control - Use hip torque
- Foot positioning

Coupling or Not:
- Couple position of leg - velocity of center of gravity
- Independent positioning of leg - velocity of each hip for yaw rotation
Quadruped Running- Attitude Control

- Hip Torque-
- Leg Thrust- Modulate differential thrust of legs providing support.
- Limit-cycle- Seesaw in a limit cycle, trusting legs act at different times in the hopping cycle

Leveling Control- Hip Torque, Leg Thrust

Rocking Control- Limit-cycle