ZMP Concept

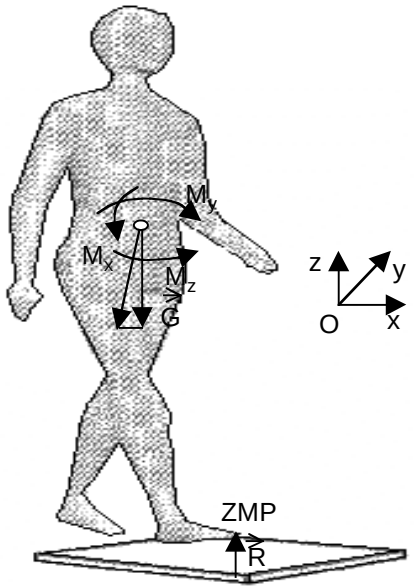
Vukobratovich, Borovac, Surla, Stokic,

“Biped Locomotion, Dynamics, Stability Control and Application”,

Springer, Berlin, 1990.

ZMP Concept

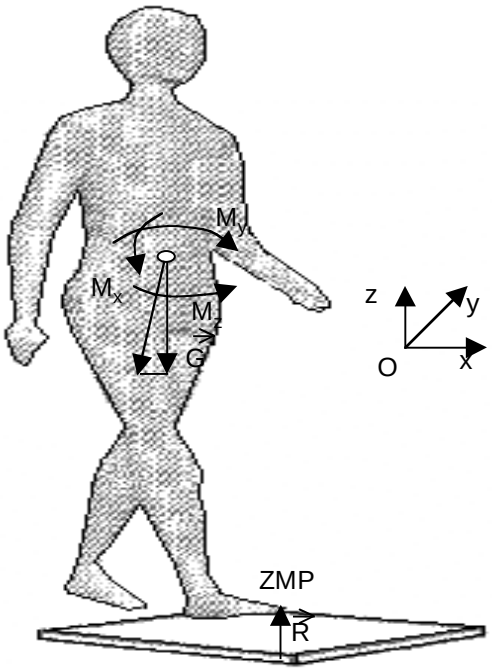
- Dynamic equilibrium and the ground reaction force R.
- M_x=0 and M_y=0. The only moment that may exist is M_z.
- Since M_x and M_y are equal to zero, this point will be the *ZMP*.



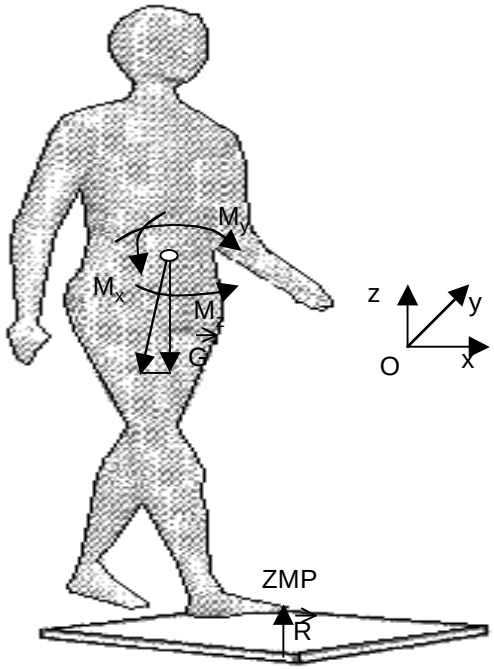
ZMP Calculation

1- Link mass concentrated in one point

2- Mass of the link is distributed.



FRP and IZMP



Active forces

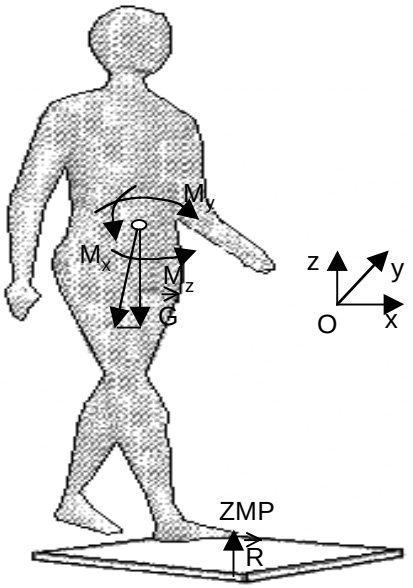
During locomotion the following active motion forces act on the body links:

G_i - gravitation force of the *i-th* link acting at the mass center C_i.

F_i - inertial force of the *i-th* link acting at the mass center C_i.

M_i - moment of the inertial force of the *i-th* link for C_i.

R- resultant ground reaction force.

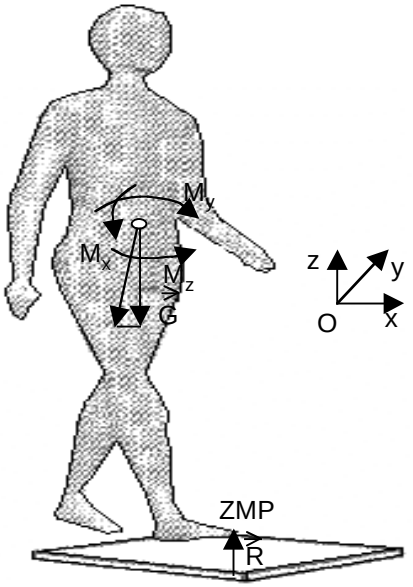


ZMP Concept

$$\vec{R} = \vec{R}_v + \vec{R}_f$$

$$\vec{M} = \vec{M}_h + \vec{M}_f$$

$$\vec{R}_v + \vec{R}_f + \sum_{j-1}^{N} \sum_{i-1}^{n_j} (\vec{F}_i + \vec{G}_i) = 0$$



$$\overrightarrow{OZMP} x \vec{R} + \sum_{j-1}^{N} \sum_{i-1}^{n_j} \overrightarrow{OC}_i \times (\vec{F}_i + \vec{G}_i) + \sum_{j-1}^{N} \sum_{i-1}^{n_j} \vec{M}_i + \vec{M}_{hZMP} + \vec{M}_{fZMP} = 0$$

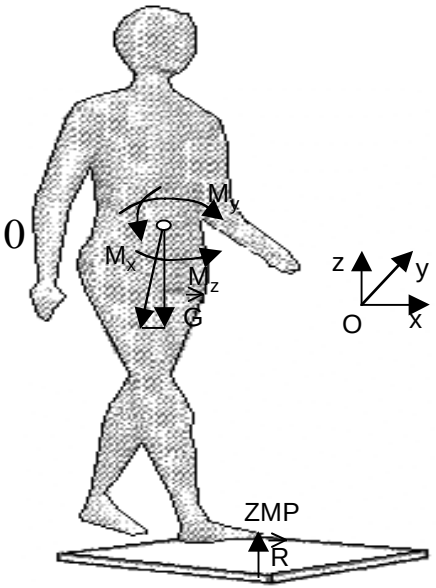
ZMP Concept

$$\overrightarrow{M} = 0$$

$$\overrightarrow{OC_i} = \overrightarrow{OZMP} + \overrightarrow{ZMPC_i}$$

$$\sum_{j-1}^{N}\sum_{i-1}^{n_j} \overrightarrow{ZMPC_i} \times (\vec{F}_i + \vec{G}_i) + \sum_{j-1}^{N}\sum_{i-1}^{n_j} \overrightarrow{M}_i + \overrightarrow{M}_{fZMP} = 0$$

$$\left(\sum_{j-1}^{N}\sum_{i-1}^{n_j} \overrightarrow{ZMPC_i} \times (\vec{F}_i + \vec{G}_i) + \sum_{j-1}^{N}\sum_{i-1}^{n_j} \overrightarrow{M}_i \right)_h = 0$$



$$\left(\overrightarrow{OZMP} \times \sum_{j-1}^{N}\sum_{i-1}^{n_j} (\vec{F}_i + \vec{G}_i) \right)_h = (\vec{R} \times \overrightarrow{OZMP})_h = \left(\sum_{j-1}^{N}\sum_{i-1}^{n_j} \overrightarrow{OC_i} \times (\vec{F}_i + \vec{G}_i) + \sum_{j-1}^{N}\sum_{i-1}^{n_j} \overrightarrow{M}_i \right)_h$$