basis for the nominal regime synthesis based on the prescribed synergy concept.

As we are dealing with mechanical systems for artificial locomotion, further considerations of applicability of the prescribed synergy will be restricted only to the problem of nominal regime synthesis for this class of mechanisms.

1.3.1. Single-support phase

The locomotion activity of a biped has certain important characteristics which have to be taken into account when an artificial motion is to be synthesized. They have already been mentioned in Section 1.1., but we are going to review them briefly and then, concentrate on the problem of artificial gait synthesis based on the prescribed synergy method.

The most important characteristic is the presence of the unpowered d.o.f. which occur only with the locomotion mechanisms, and are of basic importance for their stability as a whole. Namely, in the contact of the foot and the ground, an additional d.o.f. is formed. The mechanism would rotate around the foot edge if an undesirable situation happened, and the task of control system is to prevent this. However, each "regular" d.o.f., except the unpowered one, is powered and controlled by its own actuator and any deviation from its nominal state can be corrected only by an appropriate dynamics of the rest of the system. Because the "rest of the system" in case of locomotion mechanisms is, in fact, the whole mechanism (the unpowered d.o.f. always appears at the lowest end of the system), the dynamics of the whole locomotion mechanism is "responsible" for the behaviour of the unpowered d.o.f. If a complex mechanical structure is involved (the structure with a greater number of d.o.f.), the problem appears to be extremly inconvenient both for the nominal motion synthesis and the control of locomotion processes in the presence of disturbances.

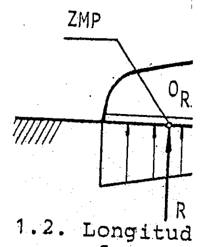
The second characteristic of such mechanisms which is of importance for the gait synthesis, is the variable mechanism structure. In the walking process, the system is alternatively supported on the one and both feet, and in this way, the configuration of the legs' kinematic chain changes from the open to a closed one. When the mechanism is supported on one leg, the situation is recognized as single-support phase, when both feet are on the ground, as double-support phase. Each of these cases gives a quite different dynamic portrait and has to be modelled separately.

A third characteristic is related to the periodical character of the mechanism motion in the walking process. The positions and velocities at the beginning and at the end of each step have to be the same, so that the walk can be performed continuously.

It is obvious that the motion of biped systems is very complex either from kinematic or dynamic point of view. The dynamics has a dominant role because of the existence of unpowered d.o.f. A further understanding of the walking process require a detailed description of the behaviour of those d.o.f. related to the motion of the whole system. The information about the forces, i.e., about the torques at mechanism's joints and the ground reaction forces under the foot (concerning both intensities and directions), are very useful in determining the system's behaviour. The following experiment can serve as proof that the human control system regulates locomotion with respect to forces. A healthy man with a temporary artificially paralysed vestibular system of natural gyroscopes can walk in a stable gait without great difficulties under the condition that a visual feedback is preserved. In such case, he is simply not able to keep a permanently steady attitude during the gait. However, the same person, with his vestibular system functioning properly but with the locomotor - muscle system of his lower extremities artificially paralysed, moves in the same way as a paralysed paraplegic person. It is quite clear that a healthy man "feels" forces, i.e., dynamic reactions, and distributes them in some manner in the form of driving torques along his skeleton activated by numerous muscle groups. Force sensing at the feet of a healthy person is a global feedback which takes care of the overall system behaviour and can be applied to artificial mechanisms, too.

Let us suppose the system is in the single-support phase and the contact with the ground is realized by the full foot (Fig. 1.2). Then, it is possible to replace all vertical elementary reaction forces by the resultant R_V. If we reduce it to the centre of supporting area, the reaction force N and moment M will be obtained. As the pressure diagram is of the same sign, the reaction force R_V can be always computed. Obviously, the ZMP in single-support phase cannot be out of the supporting area (area covered by one foot), while in double-support phase it

can be anywhere i
the ZMP can move
not, depending on



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can be anywhere inside the dashed area (Fig. 1.3). Within these areas the ZMP can move in accordance with different laws, continuously or not, depending on which gait type is performed.

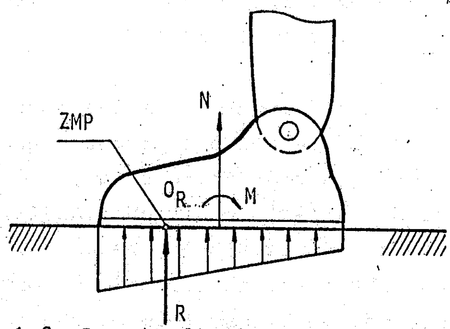

Fig. 1.2. Longitudinal distribution of pressure on the foot and ZMP position

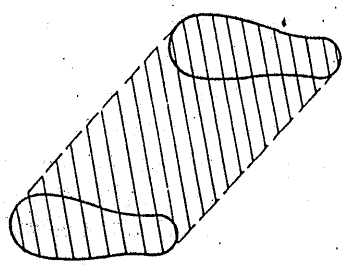

Fig. 1.3. Area of allowable positions of ZMP in double-support phase

The basic idea used in artificial synergy synthesis is that the law of change of vertical reaction and friction forces (i.e., total reaction force) under the foot is known in advance, or prescribed. The prescribed part of dynamic characteristics which, in a dynamic sense, additionally restricts the system, is named "dynamic connections". Thus, if a certain point represents the ZMP, and if the ground reaction forces $\vec{R}_V$ are reduced to it, then the moment $\vec{M}$ should be equal to zero. The vector $\vec{M}$ has always a horizontal direction and, hence, two dynamic conditions have to be satisfied: the projection of the moment on two mutually orthogonal axes X and Y in the horizontal plane should be equal to zero.

$$M_X = 0, \quad M_Y = 0 \tag{1.3.9}$$

As for the friction forces, it can be stated that their moment with respect to a vertical axis V is equal to zero:

$$M_V = 0 \tag{1.3.10}$$

The axis V can be chosen to be in any place, but if it passes through the ZMP, then the axes X, Y, and V constitute an orthogonal coordinate frame, and V will be denoted by Z.

The external forces acting on the locomotion system are the gravity, friction, and ground reaction forces. Let reduce the inertial forces and moments of inertial forces of all links to the ZMP and denote them by $\vec{F}$ and $\vec{M}_F$, respectively. The system equilibrium conditions can be derived using D'Alambert's principle, and conditions (1.3.9) can be rewritten as

$$(\vec{M}_G+\vec{M}_F)\cdot\vec{e}_X = 0, \qquad (\vec{M}_G+\vec{M}_F)\cdot\vec{e}_Y = 0 \tag{1.3.11}$$

where $\vec{M}_G$ is the total moment of gravity forces with respect to ZMP, while $\vec{e}_X$ and $\vec{e}_Y$ are unit vectors of X and Y axes of the absolute coordinate frame. Since the gravity forces are parallel to the V axis, the third equation of dynamic connections (1.3.10) becomes

$$(\vec{M}_F+\vec{\rho}\times\vec{F})\cdot\vec{e}_V = 0 \tag{1.3.12}$$

where $\vec{\rho}$ is a vector from the ZMP to the piercing point of the axis V through the ground surface; $\vec{e}_V$ is a unit vector of axis V.

Let us adopt the relative angles between two links to be the generalized coordinates and denote them by q^i. The relative (internal) angles do not depend on the choice of the absolute coordinate frame, and they are very convenient for defining the mechanism's position. Additionally, let suppose the mechanism foot rests completely on the ground, so the angle between them is zero, $q_o\equiv 0$. The inertial force $\vec{F}$ and moment $\vec{M}_F$, in a general case, can be represented in linear form of the generalized accelerations, and in quadratic form of generalized velocities

$$F^k = \sum_{i=1}^{n} a_i^k\cdot\ddot{q}^i + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^k\cdot\dot{q}^i\dot{q}^j, \qquad k=1,2,3$$

$$M_F^k = \sum_{i=1}^{n} c_i^k\cdot\ddot{q}^i + \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^k\cdot\dot{q}^i\dot{q}^j, \qquad k=1,2,3 \tag{1.3.13}$$

where the coefficients a_i^k, b_{ij}^k, c_i^k, d_{ij}^k $(k=1,2,3;\ i=1,\dots,n;\ j=1,\dots,n)$ are the functions of generalized coordinates, and F^k and M_F^k $(k=1,2,3)$ denotes projections of vectors $\vec{F}$ and $\vec{M}_F$. By introducing these expressions into (1.3.11) and (1.3.12), one obtains [1, 3]:

$$\vec{M}_G\cdot\vec{e}_X + \sum_{i=1}^{n} c_i^1\cdot\ddot{q}^i + \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^1\cdot\dot{q}^i\cdot\dot{q}^j = 0$$

$$\vec{M}_G\cdot\vec{e}_Y + \sum_{i=1}^{n} c_i^2\cdot\ddot{q}^i + \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^2\cdot\dot{q}^i\dot{q}^j = 0$$

$$\sum_{i=1}^{n} c_i^3 \ddot{q}^i + \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}^3 \dot{q}^i \dot{q}^j + \rho^X \left(\sum_{i=1}^{n} a_i^2 \ddot{q}^i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}^2 \dot{q}^i \dot{q}^j \right) -$$

$$- \rho^Y \left(\sum_{i=1}^{n} a_i^1 \ddot{q}^i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}^1 \dot{q}^i \dot{q}^j \right) = 0 \qquad (1.3.14)$$

where the superscripts X, Y denote the components in the direction of the corresponding axis.

If the locomotion system has only three d.o.f., the trajectories for all angles q^i can be computed from (1.3.14).

If, however, the mechanism is designed to perform a gait, it usually possesses more than three d.o.f. Because (1.3.14) does not contain the information about the gait type, the trajectories for the rest (n-3) coordinates should be prescribed in such a way to ensure the desired legs' trajectories. It is the easiest and most suitable way to adopt this information from measurements of human gait parameters. In this manner, the problem of legs' trajectories synthesis is avoided, since the criteria which have to be satisfied in the synthesis of a desired gait type are unknown. The trajectories for this part of the system trajectories are prescribed, while the dynamics of the rest of the system is determined in such a way to preserve the overall mechanism stability. This method, apart from the advantage of attaining a specific reduction of the system order which is neither simplification nor linearization of the mechanism, offers the possibility of realization of any desired gait type, which is of special interest in designing exoskeletons for rehabilitation of disabled persons.

Now, the set of coordinates q^i can be divided in two subsets: the first one containing all coordinates whose motion is prescribed, denoted as q^{oi}, and the second subset comprising all coordinates whose motion is to be defined using the prescribed synergy method, denoted as q^{xi}. Accordingly, the condition (1.3.14) becomes

$$\sum_{i=1}^{n} c_i^k \ddot{q}^{xi} + \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}^k \dot{q}^{xi} \dot{q}^{xj} + g^k = 0, \quad k=1,2,3 \qquad (1.3.15)$$

where c_i^k and d_{ij}^k, (k=1,2,3) are the vector coefficients dependent on q^o, and q^x, whereas vector g^k, (k=1,2,3) is a function of q^o, $\dot{q}^o$, $\ddot{q}^o$, and q^x.

In regard to the nature of legged motion of living organisms, certain repeatability conditions reflecting the feature of the gait in a stationary regime, have to be added to equations (1.3.15). Since the gait is symmetric, the repeatability conditions can be written in the form

$$q^i(0) = \pm q^i(\tfrac{T}{2}), \qquad \dot{q}^i(0) = \pm \dot{q}^i(\tfrac{T}{2})$$

where the sign depends on the physical nature of the appropriate coordinates and their derivatives; $(\frac{T}{2})$ is the duration of the one half-step period. As the motion of the prescribed part of the mechanism has been already defined (the repeatability conditions are implicitely satisfied), only for the rest of the mechanism, i.e., for the part of the mechanism whose motion has to be determined, the repeatability conditions

$$q^{xi}(0) = \pm q^{xi}(\tfrac{T}{2}), \qquad \dot{q}^{xi}(0) = \pm \dot{q}^{xi}(\tfrac{T}{2}) \qquad (1.3.16)$$

are to be added to the original set of equations describing the mechanism motion.

System (1.3.15), together with conditions (1.3.16), enables one to obtain the necessary trajectories of the coordinates q^{xi}, i.e., to carry out the compensation synergy synthesis. Accordingly, the synergy synthesis for q^{xi} coordinates is reduced to the solving of system (1.3.15) with conditions (1.3.16). For this purpose, various iterative methods for solving the boundary value problem can be applied [1, 31].

It has to be emphasized that though the equations of dynamic relations (1.3.15) have to be written only for three coordinates q^{xi}, the dynamic properties of all of the adopted anthropomorphic models are taken into account. This is a consequence of the fact that the coefficients in (1.3.15) depend on the motion of the whole mechanism. So, the mathematical model includes the inertial terms of links q^{oi} on which the prescribed synergy is imposed. In this way, the dynamics of the adopted synergy is also considered.

After the synergy synthesis is completed, the driving torques, which have to force the system to follow nominal trajectories, have to be computed. For this reason, the conditions of kinetostatic equilibrium around all joints' axes should be written and then the driving torques computed.

Dynamic connections can be set not only for the point where the ground reaction force acts on the system, but also for other joints of the mechanism. A good example is the arms motion in a walking process. Let us suppose each arm is approximated by one heavy link. Then, if there is no any specific task to be fulfilled (as, for example, the active participation of arms in the artificial gait) and if there is no any actuator at the shoulder joint, but with viscous friction involved, the behaviour of the arms is similar to that of free pendulums. It practically means that the equilibrium conditions can be written for the shoulder joints. The sum of moments of all gravity and inertial forces with respect to the shoulders joints axes, forms, along with relations (1.3.14), the additional relations of dynamic connections. During the motion, the repeatability conditions (1.3.16) have to be satisfied, too. That means the arms behaviour has to be periodic with the same period as for the other mechanism links, namely, the one-step period T. In such a case, the set q^x will contain as many q^{xi} coordinates as there are possible conditions of dynamic connections. The matrices and vectors in (1.3.15) will be of the corresponding order.

Therefore, the problem of artificial locomotion synthesis, by its nature, belongs to combined type of problem, as, for the one part of the system - motion is known, and for the rest of it - forces. The biped locomotion thus defined has certain specificities which deserve some additional explanations.

In spite of the fact that the motion of the one part of the system is prescribed (in the case we are dealing with, this is the legs' trajectories), the driving torques remain unknown. Hence, the dynamic reactions are the function of both the prescribed and compensating synergy, and they are acting on the system as unknown external forces. Because of that, it is not possible to compute the driving torques, even at the joints of prescribed trajectories. This is the reason why in the synergy synthesis a special attention has to be paid to solving these dynamical indefinitness.

The solution has been obtained by prescribing the position of the point where the ground reaction force acts on the mechanism (ZMP), to which a coordinate frame has been attached. Then, the equations of dynamic equilibrium are formed with respect to the ZMP. That means, if the legs trajectories are prescribed and the position of ZMP is chosen in advance, the sum of all moments with respect to the ZMP and axes of passive joints (shoulder joint) are equal to zero. Then, the system of differential equations with respect to the unknown part of synergy (1.3.7) becomes

$$\ddot{q}^{x} = -A_{ox}^{-1}(A_{oo}\cdot\ddot{q}^{o}+T_{o}(B[\dot{q}^{2}]+C[\dot{q}\ \dot{q}]+G)$$

and, the corresponding driving torques can be computed from (1.3.1).

In this case, the unknown synergy describes not only the motion of the compensating links for maintenance of the equilibrium with respect to the supporting point, or used for establishing the gait repeatability, but also the free motion of passive links.

1.3.2. Double-support phase

We shall consider now the double-support phase which is characterized by simultaneous contact of both mechanism's feet with the ground. In this case the kinematic chain playing the role of legs is closed, i.e., the unknown reaction forces to be determined act on its both ends. We shall describe only an approximate procedure described in monograph [1], though some other methods have been reported in the literature [5, 34].

The procedure for synergy synthesis is in the most part analogous to that for a single-support phase. Let the position of axis V be selected within the dashed area in Fig. 1.4. Then, by writing the equilibrium equations with respect to three orthogonal axes (two horizontal and one vertical) passing through the ZMP, and by setting the sum of all moments of external forces to zero, the compensating movements for the corresponding part of the body can be computed.

The next problem is how to chose the position of the axis V with respect to the ZMP. The information on ZMP and axis V is insufficient for computation of the driving torques. For this reason, it is necessary to provide some additional relations concerning the ground reaction force. These relations are concerned with the characteristics of the friction between the feet and the ground surface. The total reaction under one foot can be expressed as a sum of three reaction forces and moment components in the direction of coordinate axes. The components M_X and M_Y can be equal to zero since the diagram of the vertical forces is of the same sign. The third component, M_V, should also be equal to zero, according to the following considerations. Generally speaking, the friction forces can produce moments, but in the synergy synthesis the moment M_V should also be equal to zero. As a consequence, if moments of friction forces are generated, they should be of the opposite sign under each foot. However, in such a case these moments do not affect the system motion but only load additionally the legs drives and joints. Because of this, it is reasonable to synthesize the gait in such a way as to reduce each of these moments to zero. Consequently, it can be assumed that total moments of reaction forces under each foot are equal to zero,

$$\vec{M}_a = \vec{M}_b = 0 \tag{1.3.17}$$

where the subscripts a and b denote the left and right foot, respectively.

Now we shall discuss in more detail the total reaction force $\vec{R}$. This force has to fulfil certain relations between its horizontal and vertical component. Characteristics of the friction between the foot and the ground can be represented by a friction cone (Fig. 1.4). If the total ground reaction force is within the cone of angle 2γ, its horizontal component, opposing the sliding (i.e., the friction force) will be of sufficient intensity to prevent an unwanted horizontal motion of the supporting foot over the ground surface. This can be expressed as

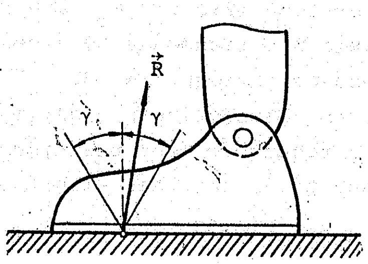

Fig. 1.4. Friction cone

$$\frac{|\vec{R}_X + \vec{R}_Y|}{|\vec{R}_V|} \leq \mathrm{tg}\gamma = \mu \tag{1.3.18}$$

where μ is the friction coefficient of the surfaces in contact. Thus we come to an important conclusion that it is reasonable to distribute the horizontal components of ground reactions per foot proportionally to the normal pressure. The vertical components are inversely proportional to the distances between the ZMP and corresponding foot. So,

$$\frac{|\vec{R}_{Va}|}{|\vec{R}_{Vb}|} = \frac{\ell_b}{\ell_a} \tag{1.3.19}$$

Then, from (1.3.18), the relation [1, 3]:

$$\frac{|\vec{T}_a|}{|\vec{T}_b|} = \frac{\ell_b}{\ell_a} \qquad (1.3.20)$$

holds for the horizontal components, where $\vec{T}_a$ and $\vec{T}_b$ are the friction forces under the corresponding foot (Fig. 1.5). On the basis of similarity of the triangles ΔOAD and ΔOBC, it can be concluded that relation (1.3.20) does not depend on the direction of the force $\vec{T}$ (i.e., the distances ℓ_a' and ℓ_b') but only on the distances between the feet, ℓ_a and ℓ_b. Thus we come to an important conclusion that, in order to have the friction forces divided in proportion to the vertical pressures, a necessary and sufficient condition is that the axis V passes through the ZMP. Then, for the synergy synthesis in double-support phase the following vector equation holds

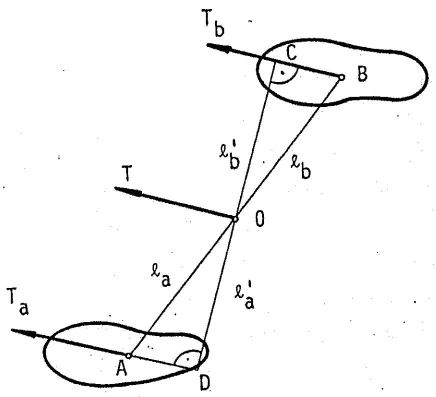

Fig. 1.5. Determination of total friction force

$$\sum_{i=1}^{n} (\vec{r}_i \times (\vec{G}_i + \vec{F}_i) + \vec{M}_i) = 0 \qquad (1.3.21)$$

where $\vec{r}_i$ is a radius vector from the ZMP to the gravity centre of the i-th link; $\vec{F}_i$ and $\vec{M}_i$ are the inertial force and the corresponding moment of the i-th link reduced to its centre of gravity.

When the synthesis of compensating laws of motion is completed, it is possible to determine total horizontal and vertical reactions

$$R_Z = -\sum_{i=1}^{n} (F_{iZ} + G_i), \qquad \vec{T} = -\sum_{i=1}^{n} (F_{iX} \cdot \vec{e}_X + F_{iY} \cdot \vec{e}_Y) \qquad (1.3.22)$$

where F_{iZ} is the projection of $\vec{F}_i$ to the vertical axis and F_{iX} and F_{iY} to axes X and Y, respectively. Here, the axis Z corresponds to a vertical axis (previously denoted by V) which passes through the ZMP. The axes X, Y, and Z with unit vectors $\vec{i}$, j and b, respectively, constitute, the absolute orthogonal coordinate frame. Furthermore the relations $\vec{T} = \vec{T}_a + \vec{T}_b$ and $\vec{R} = \vec{R}_a + \vec{R}_b$ are obvious, and they, together with the relations

$$\vec{\ell}_a \times \vec{T}_a + \vec{\ell}_b \times \vec{T}_b = 0$$
$$\vec{\ell}_a \times \vec{R}_a + \vec{\ell}_b \times \vec{R}_b = 0 \qquad (1.3.23)$$

extend the possibility of defining the vertical reactions $\vec{R}_a$ and $\vec{R}_b$, as well as the friction forces $\vec{T}_a$ and $\vec{T}_b$. The $\vec{\ell}_a$ and $\vec{\ell}_b$ are vectors from the ZMP (denoted by 0) to the centres of the corresponding supporting surfaces A and B, respectively.

When the reactions are defined according to (1.3.22) and (1.3.23), it is necessary to check out whether the inequality (1.3.18) still holds. If not, another ZMP ought to be selected and the synergy synthesis performed again as described above. When, however, (1.3.18) is satisfied, then the determination of driving torques at mechanism's joints can be worked out. For this purpose, the closed kinematic chain of legs should be broken at one end, and the corresponding reactions (for example, $\vec{R}_a$ and $\vec{T}_a$) applied. Such a situation corresponds to the case when all kinematic chains are open, and driving torques can be computed in the way already described.

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