Zero-Moment Point – Proper Interpretation

Miomir Vukobratović
Robotics Laboratory, Mihajlo Pupin Institute, 11000-Belgrade, Yugoslavia
Branislav Borovac
Faculty of Technical Sciences, 21000-Novi Sad, Trg D. Obradovi}a 6, Yugoslavia
Dragoljub Šurdilović
Fraunhofer Institute IPK-Berlin, Germany,.

Abstract:

The writing of this brief note was induced by the appearance of the two papers by Goswami(Goswami, 1999 a,b) in this journal. In these papers the author made a critical account of the ZMP concept and tried to correct (according to his opinion) the inconsistencies related to ZMP and center of pressure (CoP), considering the two notions as being identical. He also extended the area of application of ZMP to the zone being outside the foot support polygon by introducing a new notion – foot rotation indicator (FRI) point. Probably, being insufficiently informed about the works in which the ZMP concept has been introduced into practice, the author missed to notice that ZMP and CoP are not identical as, when the ZMP reaches the edge of the support polygon a perturbation moment arises and CoP does not exist any more. Besides, when determining the ZMP position in the ground plane it may happen that one obtains as a result that its position is outside the support polygon. In that case the point of support does not represent the actual (physical) ZMP but a point at which ZMP should be located (i.e. the point at which Mx =My=0) provided the support polygon is large enough to encompass the calculated point. However, as this is not a normal case and the calculated ZMP position is in the reality outside the support polygon, the calculated point represents an ***imaginary ZMP****, which means that the mechanism as a whole starts to rotate about the foot edge and overturns.*
By this paper we wanted to clarify all the pertaining ambiguities and prevent potential confusion related to the interpretation of the ZMP concept that may arise because of its imprecise formulation.

1. Introduction

Bipedal locomotion has been at the focus of researchers for decades. A basic characteristic of such a system, irrespective of its structure and complexity, is that all of its joints are powered and directly controllable except for the "joint" formed by contact of the foot and the ground. Its behavior can be controlled in an indirect way, by ensuring appropriate dynamics of the mechanism above the foot. The overall indicator of the mechanism behavior is the ground reaction force: its intensity, direction, and particularly its acting point. This point was termed *Zero-Moment Point* (*ZMP*) (Vukobratović and Juričić 1969, Juričić and Vukobratović 1972, Vukobratović and Stepanenko 1972, Vukobratović and Stepanenko 1973, Vukobratović 1973).

Recognition of the significance and role of ZMP in the biped artificial walk was a turning point in gait planning and control. The method for gait synthesis (semi-inverse method) was proposed in the two seminal works by Vukobratović and Juričić (1969) and Juričić and Vukobratović (1972), and for a long time it has remained the only method for the biped gait synthesis.

In this paper we give a brief review of the basic issues related to the original notion of ZMP, to clarify the misunderstandings caused by recently published papers (Goswami, 1999 a, b).

2. The ZMP notion

First of all we would like to clarify the basic notion and, accordingly, the name of ZMP. Let us consider the single-support phase as shown in Fig. 1, i.e. the case when only one foot is in contact with the ground (stance leg) while the other is in the swing phase, relatively passing from the back to the front position. To maintain the mechanism's dynamic equilibrium, the ground reaction force **R** should act at the appropriate point on the foot sole to balance all the forces acting on the mechanism during motion (inertial, gravitational, Coriolis and centrifugal forces and the corresponding moments), as shown in Fig. 1.

If we place the coordinate system at the point where **R** is acting (let us assume for a moment that this point is under the foot), it is clear from the equilibrium conditions that the moments acting about the horizontal axes x and y will always be equal to zero, i.e. $M_x = 0$ and $M_y = 0$. The only moment component that may exist is M_z. It is a very realistic assumption that the M_z is balanced by friction forces. Since the both moments relevant to the gait continuation (M_x and M_y) are equal zero, a natural choice to name the ground reaction force acting at this point will be zero-moment point. Any change in the locomotion dynamics will change the vector of the ground reaction force, causing simultaneous changes in its direction, intensity, and acting point (ZMP position).

The following basic ZMP definition (Vukobratović and Juričić 1969, Vukobratović *et al.* 1990) reflects the above consideration:

> ***Definition 1 (The notion of the ZMP)****: The pressure under supporting foot can be replaced by the appropriate reaction force acting at a certain point of the mechanism's foot. Since the sum of all moments of active forces with respect to this point is equal to zero, it is termed the zero-moment point (ZMP).*

In order to define ZMP in a mathematical form let us consider the dynamic model of the biped locomotion system. The robot dynamics will be modeled using the multi-body system model consisting of N chains involving the body parts. Each chain consists of n_i rigid links (i=1,...,N) interconnected with single DOF joints. During locomotion the following active motion forces act on the body links: $\vec{\mathbf{G}}_i$ - gravitation force of the i-th link acting at the mass center C_i, $\vec{\mathbf{F}}_i$ - inertial force of the i-th link acting at the mass center C_i, $\vec{\mathbf{M}}_i$ - moment of the inertial force of the i-th link for C_i, $\vec{\mathbf{R}}$ - resultant ground reaction force.

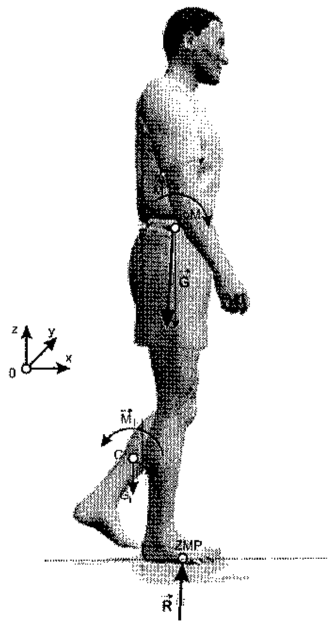

Fig. 1. Single-support phase

All active motion forces (gravitational and inertial forces and moments) can be replaced by the main resultant gravitation and inertial force and, in general case, the resultant inertial moment reduced at the body center of mass (CoM). The ground reaction force and moment can be decomposed into the vertical and horizontal components with respect to the reference frame in the following way

$$\begin{aligned} \vec{\mathbf{R}} &= \vec{\mathbf{R}}_v + \vec{\mathbf{R}}_f \\ \vec{\mathbf{M}} &= \vec{\mathbf{M}}_h + \vec{\mathbf{M}}_f \end{aligned} \qquad (1)$$

where the indices *h* and *v* denote the horizontal and vertical components respectively, while *f* indicates the friction reaction force and moment components. The following equations describe the dynamic equilibrium during the motion in the reference coordinate system if we select the ZMP as the reduction point of interest

$$\begin{aligned} &\vec{\mathbf{R}}_v + \vec{\mathbf{R}}_f + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) = 0 \\ &\overrightarrow{OZMP} \times \vec{\mathbf{R}} + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{OC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i + \vec{\mathbf{M}}_{hZMP} + \vec{\mathbf{M}}_{fZMP} = 0 \end{aligned} \tag{2}$$

where O denotes the origin of the reference frame (Fig. 1). Then, based on the ZMP definition we have

$$\vec{\mathbf{M}}_{hZMP} = 0 \tag{3}$$

Substituting the relation

$$\overrightarrow{OC_i} = \overrightarrow{OZMP} + \overrightarrow{ZMPC_i} \tag{4}$$

into the second equation of (2) and taking into account the first equation of (2) gives

$$\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{ZMPC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i + \vec{\mathbf{M}}_{fZMP} = 0. \tag{5}$$

Considering only the dynamic moment equilibrium in the horizontal ground plane (i.e. the moments that are not compensated by friction) we can write

$$\left(\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{ZMPC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i\right)_h = 0 \tag{6}$$

Substituting (4) in (6) we get

$$\left(\overrightarrow{OZMP} \times \sum_{j=1}^{N}\sum_{i=1}^{n_j}\left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right)\right)_h = \left(\vec{\mathbf{R}} \times \overrightarrow{OZMP}\right)_h = \left(\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{OC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i\right)_h \tag{7}$$

Equations (6) and (7) represent the mathematical interpretation of ZMP and provide the formalism for computing the ZMP coordinates in the ground plane. Based on (7) the notion of the ZMP can be interpreted in a way different from the basic Definition 1, e.g. the interpretation used by Dasgupta and Nakamura (1999) and Hirai *et al.* (1998) is

> ***Interpretation 1****: ZMP is the point on the walking ground surface at which the horizontal components of the resultant moment generated by active forces and moments acting on human/humanoid links are equal to zero.*

The other interpretation can be found in the work of Arakawa and Fukuda (1997)

> ***Interpretation 2****: ZMP is the point on the floor at which the moments around x and y axes generated by reaction force and moment are zero.*

All these interpretations point out certain specific aspects of the ZMP while its basic notion remains unchanged.

During the gait (let us call it *balanced gait* to distinguish it from the situation when equilibrium of the system is jeopardized and the mechanism collapses by rotating about the support polygon edge), the ground reaction force acting point can move only within the support polygon. *The gait is balanced when and only when the ZMP trajectory remains within the support area.* In the single-support phase the support polygon is identical to the foot surface. In the double-support phase, however, size of the support polygon is defined by the size of feet surface and by the distance between them (the convex hull of the two supporting feet).

3. The Difference between ZMP and Center of Pressure (CoP)

The CoP can be defined as

> ***Definition 2 (The notion of the CoP):*** *CoP represents the point on the support foot polygon at which the resultant of distributed foot ground reaction forces acts.*

In human locomotion the CoP changes during stance phase, generally moving from the heel toward a point between the first and second metatarsal heads. Indeed, it is relatively simple to demonstrate that in the considered single-support phase and for balanced stable dynamic gait equilibrium (Fig. 1) the ZMP coincides with the CoP. For this purpose let us consider again the equilibrium (2) assuming CoP as being reduction point. Let us suppose that the ZMP and CoP do not coincide. Then, according to the adopted notation, the force and moment reduced at CoP are denoted as $-\vec{\mathbf{R}}$ and $-\vec{\mathbf{M}}_{\text{CoP}}$ respectively, while the reaction force and moment are $\vec{\mathbf{R}}$ and $\vec{\mathbf{M}}_{\text{CoP}}$. Consider the equilibrium of the foot reaction forces supposing that ZMP does not coincide with CoP. For this case we can write

$$\left(\overrightarrow{\text{ZMPCoP}} \times \vec{\mathbf{R}} + \vec{\mathbf{M}}_{\text{CoP}}\right)_{\text{h}} = 0 \tag{8}$$

However, on the basis of the CoP definition for the balanced gait we have

$$\left(\vec{\mathbf{M}}_{\text{CoP}}\right)_{\text{h}} = 0 \tag{9}$$

which can only be satisfied if

$$\overrightarrow{\text{ZMPCoP}} = 0 \tag{10}$$

and, for a balanced gait, it follows that $\text{ZMP} \equiv \text{CoP}$.

Goswami (1999a, b) questioned the justification of introducing a new term (ZMP) for the already known notion in technical practice (CoP). Evidently, there are several reasons for this. While CoP is a general technical term encountered in many technical branches (e.g. fluid dynamics), the ZMP by its name expresses the essence of this point that is used exclusively in the field of biped locomotion for the gait synthesis and control. It reflects much clearer the very nature of locomotion. For example, in the biped design we can compute ZMP on the assumption that the support polygon is large enough to encompass the calculated acting point of the ground reaction force. Then we can determine the form and dimension of the foot supporting area encompassing all ZMP positions or, if needed, we can change the biped dynamic parameters, or synthesize the nominal gait and control the biped to constantly keep ZMP within the support polygon.

Furthermore, the ZMP has a more specific meaning than CoP in evaluating dynamics of the gait equilibrium. To show the specific difference between the ZMP and the CoP let us consider the dynamically unbalanced single-support situation (the mechanism as a whole rotates about the foot edge and overturns) illustrated in Fig. 2, which is characterized by a moment about CoP that could not be balanced by the foot reaction forces. In spite of the existence of a non-zero supporting area (soft foot), reaction forces cannot balance the system in such a case. As is clear from Fig. 2, in this case the CoP and the ZMP do not coincide. However, the ZMP even in such situation can be uniquely determined on the basis of its definition. Assuming that both reaction force and unbalanced moment are known, we can mathematically replace the force-moment pair with a pure force displaced from the CoP. In this situation, however, the ZMP and the assigned reaction force have a pure theorethical meaning (obviously, in such a situation, the ZMP does not coincide with the CoP) and the ZMP does not represent a physical point. However, the ZMP location outside the support area (determined by the vector $\vec{r}$ in Fig. 2) may provide very useful information for the gait balancing. The fact that ZMP is

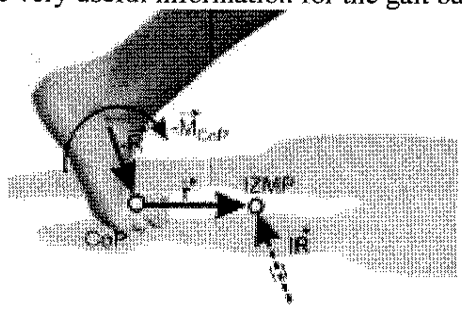

Figure 2. Action/reaction forces at CoP and ZMP (irregular case)

instantaneously on the edge or has left the support polygon indicates the occurrence of a moment that cannot be compensated for by foot reaction forces. The distance of the ZMP from the foot edge provides the measure for the unbalanced moment that tends to rotate the robot mechanism around the supporting foot and, possibly, to cause the downfall. When the system comes to such "hazardous situation" it is still possible, by means of a proper dynamic corrective action of the biped control system, to bring ZMP into the area where the equilibrium is preserved. To avoid overturning, a quite fast rebalancing by muscles or the actuator action (change of dynamic forces acting on the body) is needed. Several approaches to realization of this action have been discussed (Vukobratović *et al.* 1990).

On the basis of the above discussion it is obvious that generally the *ZMP does not coincide with the CoP*

$$ZMP \neq CoP \; . \tag{11}$$

The ZMP outside the support polygon indicates the existence of an unbalanced (irregular) gait, and does not represent a physical point related to the mechanism sole. It can be referred to as *imaginary ZMP* (*IZMP*). Three characteristic cases for the non-rigid foot in contact with the ground floor, sketched in Fig. 3, can be distinguished. In the so-called regular (balanced and repetitive) gait the ZMP coincides with CoP (Fig. 3a). If a disturbance appear, such that it brings the acting point of the ground reaction force to the foot edge, the perturbation moment will cause rotation of the complete biped system about the edge point (or a very narrow surface, under the real assumption that the sole in the shoe is not fully rigid) and its overturning. In that case we speak of the IZMP, whose imaginary position depends on the intensity of the perturbation moment (Fig. 3b). However, it is possible to realize the biped motion, for example, on the toe tips (Fig. 3c) with the aid of special shoes having pinpoint area (balletic locomotion), while keeping the ZMP position within the pinpoint area. Although it is not a regular (conventional, ordinary) gait, the ZMP still coincides with CoP.

In the double-support phase, as pointed out by Vukobratović *et al.* (1990), dynamic equilibrium is characterized by the ZMP location within the enveloping polygon between the two feet.

In this case, too, the extent of ZMP dislocation from the enveloping polygon also provides a practical measure for the unbalanced moments.

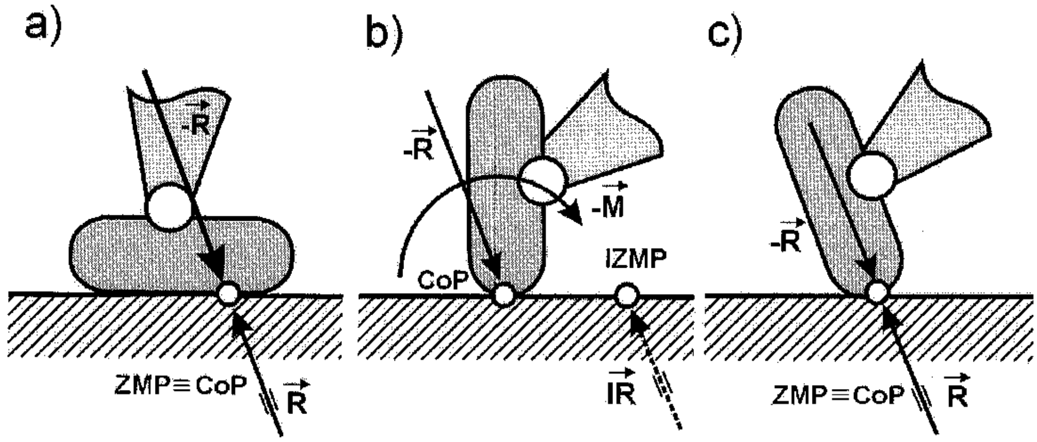

Fig. 3. The possible relative positions of ZMP and CoP: dynamically balanced gait (a), and unbalanced gait (the system as a whole rotates about the foot edge and overturns) (b), intentional foot-edge equilibrium ("balletic gait") (c)

In the previous works (Vukobratović *et al.* 1990) our attention has mainly been focused on the problems how to prevent the ZMP excursion to the positions close to the edges of the supporting polygon in the presence of various disturbances and model uncertainties. Due to limitations of the sensory and control systems, the occurrence of a ZMP at the edges of the support polygon has been considered in the past as quite critical and undesirable. Thus, there was no need for on-line computation of the IZMP location for the purpose of biped control. For these reasons the IZMP location has not gained a considerable practical importance. However, the recent development of powerful control and sensory systems, as well as rapid expansion of humanoid robots, gives a new attractive significance to the IZMP, particularly in rehabilitation robotics. The consideration of ZMP locations including also the areas outside the supporting foot sole, becomes essential for rehabilitation devices (Šurdilović and Bernhardt 2000).

4. ZMP and Foot Rotation Indicator (FRI)

Following his ZMP interpretation, Goswami (1999a,b) introduced a "novel" indicator for the gait balancing referred to as *foot rotation indicator* (FRI). According to the definition the FRI (Fig. 4) represents

> ***Definition 3 (FRI notion):*** *FRI is a point on the foot contact surface at which the resultant moment of the force/moment impressed on the foot is normal to the surface.*

At first glance the author introduced some novelty by considering the foot force components. But a very simple analysis can show that the above definition is entirely equivalent to the previously considered ZMP definition. For this purpose let us write the FRI definition in the mathematical form using the notation from Fig. 4

$$\begin{aligned}&\left(\overrightarrow{-FRIA}\times\vec{\mathbf{F}}_A-\vec{\mathbf{M}}_A+\vec{\mathbf{M}}_F+\overrightarrow{FRIC}_F\times\left(\vec{\mathbf{F}}_F+\vec{\mathbf{G}}_F\right)\right)_h=0\\&\vec{\mathbf{F}}_A=\vec{\mathbf{F}}_F+\vec{\mathbf{G}}_F+\vec{\mathbf{R}}\end{aligned}\qquad(12)$$

where the index "A" denotes the ankle joint at which the foot is separated from the rest of the body; $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{M}}_A$ are the ankle joint reactions; $\vec{\mathbf{G}}_F$ is the foot gravitation force; $\vec{\mathbf{F}}_F$ and $\vec{\mathbf{M}}_F$ are the main foot inertial force and moment; and C_F is the foot center of mass. Considering the equilibrium of the rest of the body we can write

$$\vec{\mathbf{F}}_A = -\sum_{j=1}^{N}\sum_{i=1}^{n_j}\left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \vec{\mathbf{F}}_F + \vec{\mathbf{G}}_F$$
$$\vec{\mathbf{M}}_A = -\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{AC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) - \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i + \overrightarrow{AC_F} \times \left(\vec{\mathbf{F}}_F + \vec{\mathbf{G}}_F\right) + \vec{\mathbf{M}}_F \qquad (13)$$

Substituting (13) into (12) yields

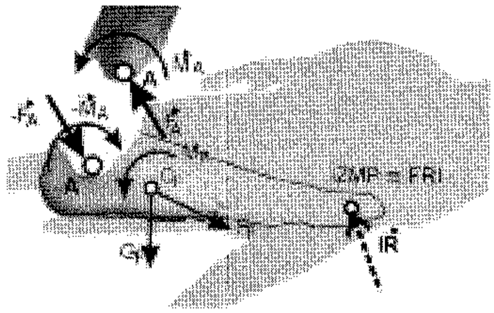

Figure 4. FRI (IZMP) and ankle/foot forces

$$\left(\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{FRIC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i\right)_h = 0 \qquad (14)$$

Comparing (14) with (6) apparently yields

$$FRI \equiv IZMP \qquad (15)$$

Hence, there is no need to introduce a new name for the already well-known and widely accepted ZMP concept.

5. Conclusion

In this brief note we attempted to clarify the confusion concerning the notions of ZMP and CoP as well as the unnecessary introduction of the FRI point that has arisen after the appearance of Goswami's papers (Goswami 1999 a,b). It has been shown that the notions of ZMP and CoP are not identical, for when the ZMP reaches the support polygon edge (either in the single- or double-support gait phase), a perturabation moment arises and CoP, by its definition, is not any more identical to ZMP. Besides, when in the synthesis of various gait types one calculates ZMP position in the ground plane it may happen that the ZMP position falls outside the support polygon. In that case the calculated point does not represent the real ZMP but a hypothetical point at which ZMP would be if the support polygon was large enough to encompass it. As this is not the real case the calculated ZMP is outside the support polygon and the point thus calculated is an *imaginary ZMP*, as that point is not actually ZMP but represents the appearance of a perturbation moment and the beginning of rotation of the mechanism as a whole about the foot edge, yielding to its fall. Of course, the distance from the IZMP to the support polygon is proportional to the intensity of the perturbation moment.

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