Zero-Moment Point – Proper Interpretation and New Applications

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1. Introduction

Biped locomotion has been at the focus of researchers for decades. Theoretical studies from various aspects have been followed by many simulations and realizations: from the simplest cases of planar mechanisms to humanoid robots, which are the most complex locomotion mechanisms constructed up to now. Irrespective of their structure and complexity, the basic characteristics of all locomotion systems are: a) the presence of *unpowered* degrees of freedom (DOF), formed by the contact of the foot with the ground surface, b) gait repeatability (symmetry), and c) regular interchangeability of the number of legs which are simultaneously in contact with the ground. During the walk, two different situations arise in sequence: the statically stable double-support phase in which the mechanism is supported on both feet simultaneously, and statically unstable single-support phase when only one foot of the mechanism is in contact with the ground, the other being transferred from the back to front position. Thus, the locomotion mechanism changes its structure during the single walking cycle from an open to a closed kinematic chain. All these circumstances have to be taken into account in the gait synthesis.

All of the biped mechanism joints are powered and directly controllable except for the "joint" formed by contact of the foot and the ground. This contact is essential for the walk realization because this is the only point at which the mechanism interacts with the environment, and the mechanism's position relative to the environment depends on the regularity of its behavior. It is often called *unpowered DOF* because in case of an improper motion, the mechanism as a whole would start to rotate about the foot edge, i.e. a "*new unpowered joint*" would appear. If such improper foot behavior happened, the position of the entire mechanism relative to the environment would be jeopardized and the mechanism would overturn.

The foot behavior cannot be controlled directly, but in an indirect way, by ensuring appropriate dynamics of the mechanism above the foot. Thus, the overall indicator of the mechanism behavior is the ground reaction force: its intensity, direction, and particularly its acting point. This point was termed *Zero-Moment Point* (*ZMP*) [1-5]. Recognition of the significance and role of ZMP in the biped artificial walk was a turning point in gait planning and control. The methods for gait synthesis (semi-inverse method) were proposed in the two seminal works by [1,2], and for a long time they have remained the only methods for the biped gait synthesis. Recently, another method has been reported [6], which, among other criteria, takes into account the gait overall indicator: the ZMP position.

The concept of ZMP has recently found attractive applications in humanoid, biped and multi-legged robots. Numerous researches have addressed the mathematical formalisms for computing the ZMP. Several algorithms for biped control and monitoring based on the ZMP concept have been proposed (for example, [7-11]). As has been demonstrated recently [12], the ZMP is also very convenient for the analysis and control of human gait in the rehabilitation robotics.

In this paper we review the basic issues related to the biped and human locomotion with a particular attention paid to the notion of ZMP because of its crucial importance for the gait analysis/synthesis and control. It will be demonstrated that the ZMP concept provides a quite useful dynamic criterion for the analysis and synthesis of the human/humanoid robot locomotion. As will be demon demonstrated, the ZMP indicates the gait balance during the entire gait cycle. Moreover, it provides a quantitative measure for the unbalanced moment about the support foot, or for the robustness ("balancing margin") of the dynamic gait equilibrium.

2. The ZMP notion

First of all we would like to clarify the notion and, accordingly, the name of ZMP. Let us consider the single-support phase as shown in Fig. 1, i.e. the case when only one foot is in contact with the ground (stance leg) while the other is in the swing phase, relatively passing from the back to the front position. To maintain the mechanism's dynamic equilibrium, the ground reaction force **R** should act at the appro-

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priate point on the foot sole to balance all the forces acting on the mechanism during motion (inertial, gravitational, Coriolis and centrifugal forces and the corresponding moments), as shown in Fig. 1.

If we place the coordinate system at the point where **R** is acting (let us assume for a moment that this point is under the foot), it is clear from the equilibrium conditions that the moments acting about the horizontal axes x and y will always be equal to zero, i.e. $M_x = 0$ and $M_y = 0$. The only moment component that may exist is M_z. It is a very realistic assumption that the friction coefficient between the ground and the foot is high enough and that M_z is balanced by friction forces. Thus, M_z will not cause foot motion and change in foot dynamics, and accordingly, it will not influence behavior of the mechanism above the foot. Since the both moments relevant to the gait continuation (M_x and M_y) are equal zero, a natural choice to name the ground reaction force acting at this point will be the Zero-Moment Point. Any change in the locomotion dynamics will change the vector of the ground reaction force, causing simultaneous changes in its direction, intensity, and acting point (ZMP).

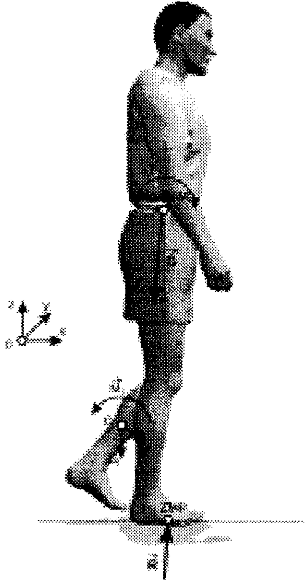

Fig. 1. Single-support phase

The following basic ZMP definition ([1,13], reflects the above consideration:

Definition 1 (The notion of the ZMP):

The pressure under supporting foot can be replaced by the appropriate reaction force acting at a certain point of the mechanism's foot. Since the sum of all moments of active forces with respect to this point is equal to zero, it is termed the zero-moment point (ZMP).

In order to define ZMP in a mathematical form let us consider the dynamic model of the human, i.e. of a humanoid or biped robot (the following analysis can be applied to all these systems). The human/humanoid dynamics will be modeled using the multi-body system model consisting of N chains involving the body parts (head, arms, legs, trunk and pelvis). Each chain consists of n_i-links (i=1,...,N) interconnected with single DOF joints (the multiple DOF joints are decomposed into the single ones). For the sake of simplicity let us consider the rigid body model that is a relatively good approximation of the humanoid dynamics, though it represents a very idealized model of the human gait. The multi-DOF structure of the human "locomotion mechanism", joint flexibility, and structural and behavioral complexity of the foot, support the realization of very dynamic gait patterns that are difficult to achieve with the existing humanoid systems [14].

During locomotion the following active motion forces act on the body links: $\vec{\mathbf{G}}_i$ - gravitation force of the i-th link acting at the mass center C_i, $\vec{\mathbf{F}}_i$ - inertial force of the i-th link acting at the mass center C_i, $\vec{\mathbf{M}}_i$ - moment of the inertial force of the i-th link for C_i, $\vec{\mathbf{R}}$ - resultant ground reaction force,

All active motion forces (gravitational and inertial forces and moments) can be replaced by main resultant gravitation and inertial force and, in general case, resultant inertial moment reduced at body center of mass (CoM). The ground reaction force and moment can be decomposed into the vertical and horizontal components with respect to the reference frame. The horizontal reaction force represents the friction force that is essential for preserving the contact between the foot and the ground. The vertical reaction moment represents the moment of the friction reaction forces reduced at an arbitrary point *P*. We will assume a stable foot-floor contact without sliding. This means that the static friction forces compensate for the corresponding dynamic body reaction forces. Accordingly, the vertical reaction force and horizontal reaction moment components represent the dynamic reaction forces that are not compensated by the friction. The decomposition will be presented in the following form

$$\vec{\mathbf{R}} = \vec{\mathbf{R}}_v + \vec{\mathbf{R}}_f, \quad \vec{\mathbf{M}} = \vec{\mathbf{M}}_h + \vec{\mathbf{M}}_f \qquad (1)$$

where the indices *h* and *v* denote the horizontal and vertical components respectively, while *f* indicates the friction reaction force and moment components. Let us select the ZMP as the reduction point of interest, i.e. P = ZMP. Then the following equations express the dynamic equilibrium during the motion in the reference coordinate system

$$\vec{\mathbf{R}}_v + \vec{\mathbf{R}}_f + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) = 0$$

$$\overrightarrow{OZMP} \times \mathbf{R} + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{OC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \qquad (2)$$

$$+ \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i + \vec{\mathbf{M}}_{hZMP} + \vec{\mathbf{M}}_{fZMP} = 0$$

where O denotes the origin of the reference frame (Fig. 1). Then, based on the ZMP definition we have

$$\vec{\mathbf{M}}_{hZMP} = 0 \qquad (3)$$

Substituting the relation

$$\overrightarrow{OC_i} = \overrightarrow{OZMP} + \overrightarrow{ZMPC_i} \qquad (4)$$

into the second equation of (2) and taking into account the first equation of (2) gives

$$\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{ZMPC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i + \qquad (5)$$

$$+ \vec{\mathbf{M}}_{fZMP} = 0$$

Considering only the dynamic moment equilibrium in the horizontal ground plane (i.e. the moments that are not compensated by friction) we can write

$$\left(\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{ZMPC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i\right)_h = 0 \qquad (6)$$

Substituting (4) in (6) yields

$$\left(\overrightarrow{OZMP} \times \sum_{j=1}^{N}\sum_{i=1}^{n_j}\left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right)\right)_h = \left(\vec{\mathbf{R}} \times \overrightarrow{OZMP}\right)_h =$$

$$= \left(\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{OC_i} \times \left(\vec{\mathbf{F}}_i + \vec{\mathbf{G}}_i\right) + \sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i\right)_h \qquad (7)$$

The equations (6) and (7) represent the mathematical interpretation of ZMP and provide the formalism for computing the ZMP coordinates in the horizontal ground plane. Based on (7) the notion of the ZMP can be interpreted in a different way compared to the basic Definition 1, e.g. the interpretation used in [14,15] is

Interpretation 1:
ZMP is the point on the walking ground surface at which the horizontal components of the resultant moment generated by active forces and moments acting on human/hu-manoid links are equal to zero.

The other interpretation can be found in [6]

Interpretation 2:
ZMP is the point on the floor at which the moments around x and y axes generated by reaction force and moment are zero.

All these interpretations point out specific aspects of the ZMP while its basic notion remains unchanged.

As already mentioned, the one-step cycle consists of the single- and double-support phase, taking place in sequence. A basic difference between these elemental motion phases is that during the motion in the single-support phase the position of the free foot is not fixed relative to the ground, while in the double-support phase the positions of the both feet are fixed. But, from the ZMP's point of view situation is identical - in both cases ZMP should remain within the support polygon in order to maintain balance. During the gait (let us call it *balanced gait* to distinguish it from the situation when equilibrium of the system is jeopardized and the mechanism collapses by rotating about the support polygon edge), the ground reaction force acting point can move only within the support polygon. *The gait is balanced when and only when the ZMP trajectory remains within the support area.* In this case, the system dynamics is perfectly balanced by the ground reaction force and the overturning will not occur. In the single-support phase the support polygon is identical to the foot surface. In the double-support phase, however, size of the support polygon is defined by the size of feet surface and by the distance between them (the convex hull of the two supporting feet).

This ZMP concept is primarily related to the gait dynamics, however it can also be applied to consider static equilibrium, when the robot maintains a certain posture. The only difference is in the forces inducing the ground reaction force vector: in the static case there is only the mechanism weight, while the gait involves dynamic forces too. Accordingly, when equilibrium of a static posture (the mechanism is "frozen" in a certain posture and no gait is performed) is considered, the vertical projection of total active force acting at the mass center must be within the support polygon. This is a well-known condition for the static equilibrium.

3. The Difference between ZMP and Center of Pressure (CoP)

At first glance [16] one can see from the above analysis that ZMP is apparently equivalent to the center of pressure (CoP), representing the application point of the ground reaction forces (GRF) that is commonly applied in the humah gait analysis based on the force platfom measurements. The CoP can be defined as

Definition 2 (The notion of the CoP):
CoP represents the point on the support foot polygon at which the resultant of distributed foot ground reaction forces acts.

The CoP is commonly used in the human gait analysis based on the force platform or pressure mats measurements. In human locomotion the CoP changes during stance phase generally moving from the heel toward a point between the first and second metatarsal heads. Indeed it is relatively simple to demonstrate that in the considered single-support phase and for balanced stable dynamic gait equilibrium (Fig. 1) the ZMP coincides with the CoP. For this purpose let us consider again the equilibrium (2) assuming CoP as being the reduction point, P=CoP. Let us suppose that ZMP and CoP do not coincide. Then according to the adopted notation, the force and moment reduced at CoP are denoted as $-\vec{\mathbf{R}}$ and $-\vec{\mathbf{M}}_{CoP}$ respectively, while the reaction force and moment are $\vec{\mathbf{R}}$ and $\vec{\mathbf{M}}_{\mathrm{CoP}}$. Consider the equilibrium of the foot reaction forces supposing that ZMP does not coincide with CoP. For this case we can write

$$\left(\overrightarrow{\mathrm{ZMPCoP}}\times\vec{\mathbf{R}}+\vec{\mathbf{M}}_{\mathrm{CoP}}\right)_{\mathrm{h}}=0 \tag{8}$$

However, on the basis of CoP definition for the balanced gait we have

$$\left(\vec{\mathbf{M}}_{CoP}\right)_h=0 \tag{9}$$

which can only be satisfied if

$$\overrightarrow{\boldsymbol{ZMPCoP}}=0 \tag{10}$$

and it follows that $\mathrm{ZMP}\equiv\mathrm{CoP}$.

Goswami [17,18] questioned the justification of introducing a new term (ZMP) for the already known notion in technical practice (CoP). Evidently, there are several reasons for this. While CoP is a general technical term encountered in many technical branches (e.g. fluid dynamics), the ZMP by its name expresses the essence of this point that is used exclusively in the field of biped locomotion for the gait synthesis and control. It reflects much clearer the very nature of locomotion. For example, in the biped design we can compute ZMP on the assumption that the support polygon is large enough to encompass the calculated acting point of the ground reaction force. Then we can determine the form and dimension of the foot supporting area encompassing all ZMP points or, if needed, we can change the biped dynamic parameters, or synthesize the nominal gait and control the biped to constantly keep ZMP within the support polygon.

Furthermore, the ZMP has a more specific meaning than CoP in evaluating dynamics of the gait equilibrium. To show the specific difference between the ZMP and the CoP [16] let us consider the dynamically unbalanced single-support situation (the mechanism as a whole rotates about the foot edge and overturns) illustrated in Fig. 2, which is characterized by a moment about CoP that could not be balanced by the sole reaction forces. The reaction moment that can be generated between the foot and the ground is limited due to the unilateral contact between each sole and the floor. The intensity of balancing moments depends on the foot dimension. Obviously, for a person with larger sole it is easier to balance the gait. The dynamic motion moments in specific cases may exceed the limit, causing the foot to leave the ground. In spite of the existence of a non-zero supporting area (soft human/humanoid foot), reaction forces cannot balance the system in such a case. The way in which this situation in human/humanoid gait could practically occur will be considered later. As is clear from Fig. 2, in this case the CoP and the ZMP do not coincide. Using the analogy with fluid dynamics even for this case we could determine the CoP as the center of pressure distribution (e.g. obtained by a pressure mate). It should be mentioned that in regular human gait, in quite a dynamic transition phase (e.g. heel-strike and toe-off), it is quite difficult and uncertain to estimate CoP on the basis of force plate measurements.

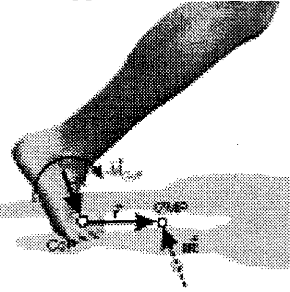

Figure 2. Action/reaction forces at CoP and ZMP (irregular case)

However, the ZMP even in the case illustrated in Fig. 2 can be uniquely determined on the basis of its definition. Assuming that both reaction force and unbalanced moment are known, we can mathematically replace the force-moment pair with a pure force displaced from the CoP. In this situation, however, the ZMP and the assigned reaction force have a pure mathematical/mechanical meaning (obviously, the ZMP does not coincide with the CoP in such a situation) and the ZMP does not represent a physical point. However, the ZMP location outside the support area (determined by the vector $\vec{\mathbf{r}}$ in Fig. 2) provides very useful information for the gait balancing. The fact that ZMP instantaneously is on the edge or has left the support polygon indicates the occurrence of an unbalanced moment that cannot be compensated for by foot reaction forces. The distance of ZMP from the foot edge provides the measure for the unbalanced moment that tends to rotate the human/humanoid around the supporting foot and, possibly, to cause the downfall. When the system comes to such "hazardous situation" it is still possible by means of a proper dynamic corrective action of the biped control system to bring ZMP into the area where the equilibrium is preserved. To avoid this, aquite fast rebalancing by muscles or actuator actions (change of dynamic forces acting on the body) is needed. Several approaches to realization of this action have been discussed in [13].

On the basis of the above discussion it is obvious that generally [16] the *ZMP does not coincide with the CoP*

$$\boldsymbol{ZMP \neq CoP} \tag{11}$$

As is well known, the *CoP may never leave* the support polygon. However, the ZMP even in the single-support gait phase *can leave* the polygon of the supporting foot, when the gait is not dynamically balanced by foot reaction forces, e.g. in the case of a non-regular gait (even in case of a degenerative gait). Hence ZMP provides a more convenient dynamic criterion for the gait analysis and synthesis.

As mentioned above, the ZMP outside the support polygon indicates an unbalanced (irregular) gait [16] and does not represent a physical point related to the mechanism sole. It can be referred to as *imaginary ZMP* (*IZMP*). Three characteristic cases for the non-rigid foot in contact with the ground floor, sketched in Fig. 3, can be distinguished. In the so-called regular (balanced and repetitive) gait the ZMP coincides with CoP (Fig. 3a). If a disturbance appear, such that it brings the acting point of the ground reaction force to the foot edge, the perturbation moment will cause rotation of the complete biped locomotion system about the edge point (or a very narrow surface, under the real assumption that the sole in the shoe is not fully rigid) and its overturning (Fig. 4). In that case we speak of imaginary ZMP (IZMP), whose imaginary position depends on the intensity of the perturbation moment (Fig. 3b). However, it is possible to realize the biped motion, for example, on the toe tips (Fig. 3c) with special shoes having pinpoint area (balletic locomotion), while keeping the ZMP position within the pinpoint area. Although, it is not again a regular (conventional, ordinary) gait, the ZMP also coincides with CoP.

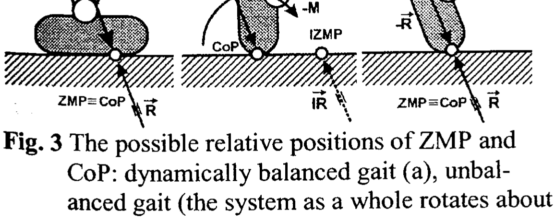

Fig. 3 The possible relative positions of ZMP and CoP: dynamically balanced gait (a), unbalanced gait (the system as a whole rotates about the foot edge and overturns) (b), and intentional foot-edge equilibrium ("balletic locomo tion") (c)

As mentioned above, because of the foot elasticity and the very complex form of supporting area the ZMP displacements outside the "safe zone" (Fig. 2) in human locomotion are much morecomplex and difficult to model. Even in a "regular" human gait ZMP leaves the support polygon very dynamically during the transition from the single- to double-support phase, providing thus a very smooth dynamic locomotion. The implementation of such gait patterns in humanoids with simple rigid foot is not practically possible [14].

In the double-support phase, and even more during transition from the single to the double phase, the ZMP leaves the foot-supporting polygon. As has been pointed out [13] stable dynamic equilibrium in the double-support phase is characterized by the ZMP location within the enveloping polygon between the two feet. In this case, the extent of ZMP dislocation from the enveloping polygon also provides a practical measure for the unbalanced mo-ments. In the previous works [13] our attention has mainly been focused on the problems of biped design and nominal motion synthesis, as well as stability analysis and biped dynamic control that will prevent the ZMP excursions close to the edges of supporting polygon in spite of various disturbances and model uncertainties. Due to limitations of the sensory and control systems, the occurrence of a "new unpowered joint" (ZMP at the edges of the support polygon) has been considered in the past as quite critical and undesirable. Hence, the situation when ZMP can arbitrarily be located in the foot plane was primarily practical in designing the biped foot dimensions and nominal motion synthesis. In the critical situation, when the ZMP approaches critical areas or even abandons the support polygon balancing is foremost focused on compensating for the unbalanced dynamic moment using the posture control. One way of overcoming such critical situation is to switch to a new nominal trajectory prepared in advance [5] that is closest to the momentary system state. These nominals are synthesized to bring the system back to the stationary state and enable gait continuation. To do this it is not necessary to have information about exact intensity of the disturbance moment. For such an approach (which is, actually, very close to the human behavior in similar situations) it suffices to detect the occurrence of such hazardous situation. Thus, there is no need for on-line computation of the IZMP location for the purpose of biped control. For these reasons the IZMP location has not gained more considerable practical importance. However, the recent development of powerful control and sensory systems, as well as fast expansion of humanoid robots, gives a new attractive significance to the IZMP, particularly in rehabilitation robotics. The consideration of ZMP locations, also including the areas outside the supporting foot sole, becomes essential for rehabilitation devices [12].

In the future, it can be expected that gait performance of artificial locomotion mechanisms will be much closer to that of humans. Essential for this is the development of compliant humanoid feet and force/impedance interaction control algorithms. There are some most challenging subjects for the further humanoid development. Probably, it is not too optimistic to expect that in the close future new humanoids will significantly approach human balancing capabilities.

4 ZMP and Foot Rotation Indicator (FRI) point

Following his ZMP interpretation, Goswami introduced a "novel" indicator for the human/humanoid gait balancing referred to as *foot rotation indicator* (FRI). According to the definition [15,16] the FRI (Fig. 5) represents

Definition 3 (FRI notion):

FRI is a point on the foot contact surface at which the resultant moment of the force/moment impressed on the foot is normal to the surface.

Indeed this definition is identical to the definition of the ZMP! At first glance the author introduced some novelty by considering the foot force components. But a very simple analysis can show that the above definition is totally equivalent to the previously considered ZMP definition. For this purpose let us write the FRI definition in the mathematical form using the notation from Fig. 4

$$\begin{gathered}\left(\overrightarrow{-\boldsymbol{FRIA}}\times\vec{\mathbf{F}}_A-\vec{\mathbf{M}}_A+\vec{\mathbf{M}}_F+\overrightarrow{\boldsymbol{FRIC}}_F\times\left(\vec{\mathbf{F}}_F+\vec{\mathbf{G}}_F\right)\right)_h=0\\ \vec{\mathbf{F}}_A=\vec{\mathbf{F}}_F+\vec{\mathbf{G}}_F+\vec{\mathbf{R}}\end{gathered}\qquad(12)$$

where the index "A" denotes the ankle joint at which

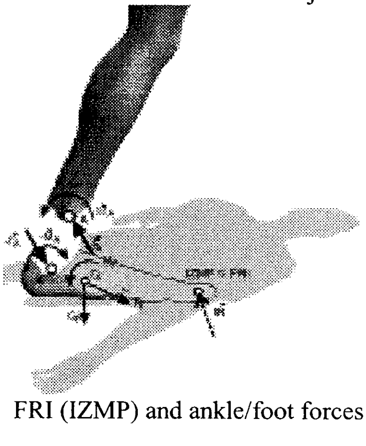

Figure 4. FRI (IZMP) and ankle/foot forces

the foot is separated from the rest of the body, $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{M}}_A$ are the ankle joint reactions, $\vec{\mathbf{G}}_F$ is the foot gravitation force, $\vec{\mathbf{F}}_F$ and $\vec{\mathbf{M}}_F$ are the main foot inertial force and moment and $\mathrm{C_F}$ is the foot center of mass. Considering the equilibrium of the rest of the body we can write

$$\begin{aligned}\vec{\mathbf{F}}_A&=-\sum_{j=1}^{N}\sum_{i=1}^{n_j}\left(\vec{\mathbf{F}}_i+\vec{\mathbf{G}}_i\right)+\vec{\mathbf{F}}_F+\vec{\mathbf{G}}_F\\ \vec{\mathbf{M}}_A&=-\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{AC}_i\times\left(\vec{\mathbf{F}}_i+\vec{\mathbf{G}}_i\right)-\sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i+\\ &\quad+\overrightarrow{AC}_F\times\left(\vec{\mathbf{F}}_F+\vec{\mathbf{G}}_F\right)+\vec{\mathbf{M}}_F\end{aligned}\qquad(13)$$

$$\left(\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{\boldsymbol{FRIC}_i}\times\left(\vec{\mathbf{F}}_i+\vec{\mathbf{G}}_i\right)+\sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i\right)_h=0\qquad(14)$$

Comparing (14) with (6) apparently yields

$$\boldsymbol{FRI}\equiv\boldsymbol{IZMP}\qquad(15)$$

Obviously, Goswami introduced a new name for the already well-known and widely accepted ZMP concept.

5. New application of the ZMP concept in human gait restoration

Within the Wisa-ROMED project endowed by the Fraunhofer Community, one of the project demonstration systems (RehaRob) represents the first application of the ZMP concept for rehabilitation and control of the human gait using treadmill training with partial body-weight bearing (PWB). This methodology has recently been successfully applied for the gait recovery of stroke patients [19].

The global RehaRob architecture consists of the following main components: active weight relieving robotic system-wire robot, harness system (patient interface), treadmill and/or lifting system axial motion device supporting the repetitive motion progression, sensory system (motion camera, insole pressure sensors, force sensors, inclinometers, wire position sensors), control system, rehabilitation planning and programming system (user interface), AR i.e. VR system providing visual feedback, safety computer control and mechanical system providing exception-handling functions.

The robot wires are connected to the trunk and pelvis at optimized attachment points (in the actual system under development totally 10 wires are applied). By this means the robot exerts the external active forces upon the trunk and pelvis in order to reduce the weight on lower limb (reaction force) and to balance the posture, thus essentially supporting the gait. The redundant number of wires is needed in order to ensure the tension in all wires independent of dynamic loads (rigid trunk-pelvis system connected with a spherical joint has 9 DOFs).

The practical determination of ZMP in human walking is coupled with difficulties concerning the measurement of joint motion and modeling of limbs dynamics. Several practical algorithms were proposed to estimate CoM based on foot forces or camera motion measurements [20]. Some of the proposed methods also use the human dynamic walking models. However, the range of CoM motion is quite small and this concept is very difficult to apply for the gait involve various algorithms for CoP estimation. However irregular situations are quite frequent and ZMP indicates these situations and the wire robot system can exert external forces to prevent patient downturn.

The application of the human motion ZMP trajectory for controlling a biped robot has recently been proposed [14]. For the body modeling the RehaRob uses a rigid model of the human developed in MATLAB (MatMan) with 37 DOFs. Apparently, the ZMP is in a period of time within the stationary stance foot supporting area, while in the remaining time leaves this area following moving swing foot. As mentioned, in a stabile gait the ZMP remains within the enveloping area constrained by foot projections on the ground. Different to the ZMP model of human/humanoid walking (7), the model of the RehaRob system includes additional wire active forces affecting the equilibrium conditions (2) according to

$$\vec{\mathbf{R}}+\sum_{j=1}^{N}\sum_{i=1}^{n_j}\left(\vec{\mathbf{F}}_i+\vec{\mathbf{G}}_i\right)+\sum_{k=1}^{S}\vec{\mathbf{S}}_k=0$$

$$\begin{aligned}(\overrightarrow{OZMP}\times\vec{\mathbf{R}}+\sum_{j=1}^{N}\sum_{i=1}^{n_j}\overrightarrow{OC_i}\times\left(\vec{\mathbf{F}}_i+\vec{\mathbf{G}}_i\right)+\\ +\sum_{j=1}^{N}\sum_{i=1}^{n_j}\vec{\mathbf{M}}_i+\sum_{k=1}^{S}\overrightarrow{OS_k}\times\vec{\mathbf{S}}_k)_h=0\end{aligned} \qquad (16)$$

where S_k are the attachment points on the trunk and pelvis, while $\vec{\mathbf{S}}_k$ are the wire forces that are measured using force sensors. The sensors provide the intensity of the wire tension force, while the force vectors are computed using the model of the wire robot. In order to cope with the modeling errors the external sensors measuring trunk posture are used.

It is apparent from (16) that the wire forces allow control of both the body reaction and ZMP location. However, the practical problem is still the estimation inertial forces and moments, also occurring in the dynamic model (16) and influencing the ZMP temporary location. For fairly accurate computation of inertia properties of human limbs based on the general data (human weight and height) several practical statistical methods have been developed [21]. However, as is known, these approximate methods are coupled with the errors in the ZMP estimation.

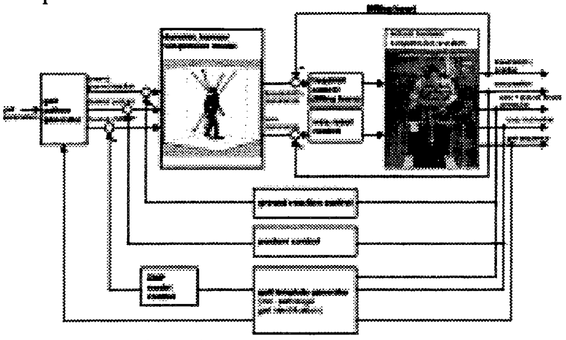

Figure 5. RehaRobot control concept

The motion of the relevant upper and lower limbs (e.g. knee) can be measured using relatively cheap sensors, while the trunk and pelvis position in the RehaRob system are directly measured and controlled using both wires and body sensors. To cope with the model inaccuracies and the ZMP estimation errors, the RehaRob system implements relatively complex control structure closing several control loops (Fig. 5) around reaction force, gait posture, as well using the internal wire robot and treadmill control.

This control scheme is similar to the recent humanoid control approaches proposed [15,21]. The principal differ-ence is that the RehaRob control system utilizes external wire forces instead of cooperative dynamic body links motion. The control scheme includes the basic gait pattern (initial contact, stance, swing, single limb) generator, which, on the basis of the actual captured gait state and required weight suspension percentage generates on-line the desired ground reaction, as well as nominal posture and ZMP location data. These values are compared with the measured (i.e. estimated) ones and the control feedback is closed around the dynamic human gait and wire robot models providing the inputs to the internal wire and treadmill control loops. Basically, the local controllers control the wire robot system so that the posture can support and follow joint motion of the desired walking pattern. The gait balance is realized utilizing ZMP and posture controls for the generated pattern. The gait templates generator involves the knowledge about the subject abnormal gait, as well as emergency and exception handling strategies (e.g. to compensate for 100% weight upon stance leg if the condition for the single-leg support are not available, for example due to improper ankle or knee joints position etc.).

The ZMP location is practically controlled by dynamic trunk and pelvis posture control and by wire forces. The patient moves the lower limbs according to own possibilities and wishes, while the system captures the lower limb motion, estimates the actual gait phase and predicts the next one, and supports the patient by controlling the ZMP and reaction force. In order to support the patient's own initiative during the locomotion training the wire robot control includes the compliance (i.e. impedance) interaction control.

6. Conclusion

This paper is dealing with the notion and role of ZMP in artificial biped locomotion as well as in new applications for motoric gait rehabilitation [22]. It was demonstrated that the ZMP concept provides a quite useful dynamic criterion for the characterization and monitoring of the human/humanoid robot locomotion. As demonstrated, the ZMP indicates the static and dynamic gait equilibrium during the entire gait cycle. Moreover it provides a quantitative measure for the unbalanced gait and for the robustness of gait equilibrium. The concept of ZMP is also quite useful for the analysis and control of the human gait in rehabilitation robotics. This was demonstrated on the example of the new wire robot system RehaRob designed to support dynamic and programmable weight bearing and gait restoration.

The relationship between ZMP and CoP was also considered.

7. References

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