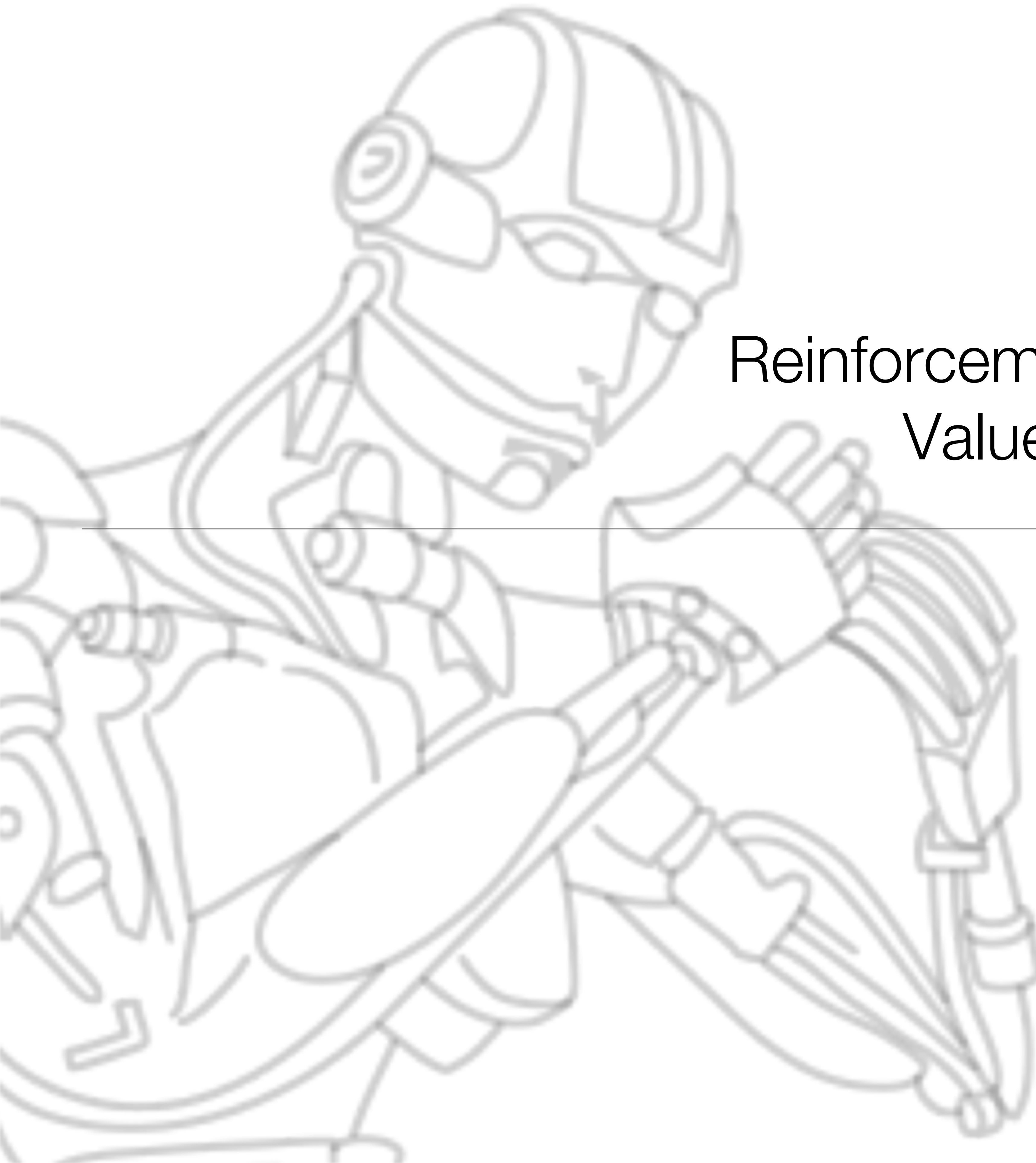
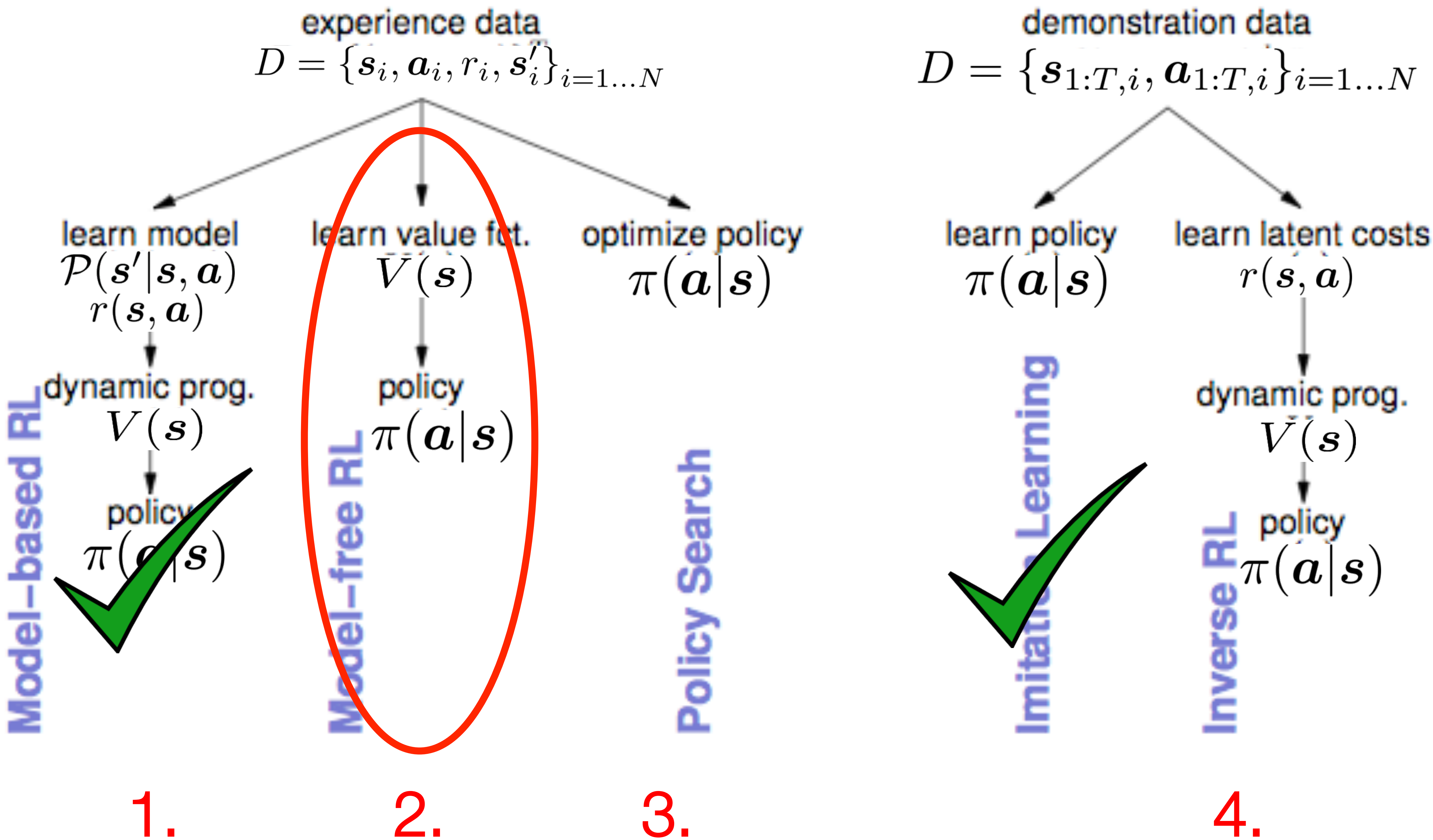


Reinforcement Learning Part 2
Value Function Methods

Jan Peters
Gerhard Neumann

The Bigger Picture: How to learn policies



Purpose of this Lecture

Often, learning a good model is too hard

- ➡The optimization inherent in optimal control is prone to model errors, as the controller may achieve the objective only because model errors get exploited
- ➡Optimal control methods based on linearization of the dynamics work only for moderately non-linear tasks
- ➡Model-free approaches are needed that do not make any assumption on the structure of the model

Classical Reinforcement Learning:

- ➡Solve the optimal control problem by learning the value function, not the model!

Outline of the Lecture

1. **Quick recap of dynamic programming**
2. **Reinforcement Learning with Temporal Differences**
3. **Value Function Approximation**
4. **Batch Reinforcement Learning Methods**
   Least-Squares Temporal Difference Learning
   Fitted Q-Iteration
5. **Robot Application:** Robot Soccer

Final Remarks

Markov Decision Processes (MDP)

Classical reinforcement learning is typically formulated for the infinite horizon objective

Infinite Horizon: maximize **discounted accumulated reward**

$$J_{\boldsymbol{\pi}} = \mathbb{E}_{\mu_0, \mathcal{P}, \pi} \left[\sum_{t=0}^{\infty} \gamma^t r(\boldsymbol{s}_t, \boldsymbol{a}_t) \right]$$

$0 \leq \gamma < 1$ … discount factor

Trades-off long term vs. immediate reward

Value functions and State-Action Value Functions

Refresher**:** Value function and state-action value function can be computed iteratively

$$V^\pi(\boldsymbol{s}) = \mathbb{E}_\pi\Big[r(\boldsymbol{s},\boldsymbol{a}) + \gamma \mathbb{E}_{\mathcal{P}}\left[V^\pi(\boldsymbol{s}')\right] \Big| \boldsymbol{s}\Big]$$

$$= \int \pi(\boldsymbol{a}|\boldsymbol{s})\Big(r(\boldsymbol{s},\boldsymbol{a}) + \gamma \int \mathcal{P}(\boldsymbol{s}'|\boldsymbol{s},\boldsymbol{a}) V^\pi(\boldsymbol{s}') d\boldsymbol{s}'\Big) d\boldsymbol{a}$$

$$Q^\pi(\boldsymbol{s},\boldsymbol{a}) = r(\boldsymbol{s},\boldsymbol{a}) + \gamma \mathbb{E}_{\mathcal{P},\pi}\Big[Q^\pi(\boldsymbol{s}',\boldsymbol{a}') \Big| \boldsymbol{s},\boldsymbol{a}\Big]$$

$$= r(\boldsymbol{s},\boldsymbol{a}) + \gamma \int \mathcal{P}(\boldsymbol{s}'|\boldsymbol{s},\boldsymbol{a}) \int \pi(\boldsymbol{a}'|\boldsymbol{s}') Q^\pi(\boldsymbol{s}',\boldsymbol{a}') d\boldsymbol{a}' d\boldsymbol{s}'$$

Finding an optimal value function

Bellman Equation of optimality

$$V^*(\boldsymbol{s}) = \max_{\boldsymbol{a}} \left(r(\boldsymbol{s}, \boldsymbol{a}) + \gamma \mathbb{E}_{\mathcal{P}} \left[V^*(\boldsymbol{s}') \middle| \boldsymbol{s}, \boldsymbol{a} \right] \right)$$

⇨ Iterating the Bellman Equation converges to the optimal value function V^* and is called **value iteration**

Alternatively we can also **iterate Q-functions…**

$$Q^*(\boldsymbol{s}, \boldsymbol{a}) = r(\boldsymbol{s}, \boldsymbol{a}) + \gamma \mathbb{E}_{\mathcal{P}} \left[\max_{\boldsymbol{a}'} Q^*(\boldsymbol{s}', \boldsymbol{a}') \middle| \boldsymbol{s}, \boldsymbol{a} \right]$$

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Final Remarks

Value-based Reinforcement Learning

Classical Reinforcement Learning

Updates the value function based on samples

$$\mathcal{D} = \{\boldsymbol{s}_i, \boldsymbol{a}_i, r_i, \boldsymbol{s}'_i\}_{i=1\ldots N}$$

We do not have a model and we do not want to learn it

Use the samples to update Q-function (or V-function)

Lets start simple:

Discrete states/actions ➡ Tabular Q-function

Temporal difference learning

Given a transition (s_t, a_t, r_t, s_{t+1}), we want to update the V-function

- Estimate of the current value: $V(s_t)$
- 1-step prediction of the current value: $\hat{V}(s_t) = r_t + \gamma V(s_{t+1})$
- 1-step prediction error (called temporal difference (TD) error)

$$\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$$

Update current value with the temporal difference error

$$V_{\text{new}}(s_t) = V(s_t) + \alpha \delta_t = (1 - \alpha) V(s_t) + \alpha (r_t + \gamma V(s_{t+1}))$$

Temporal difference learning

The **TD error**

$$\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$$

compares the **one-time step lookahead prediction**

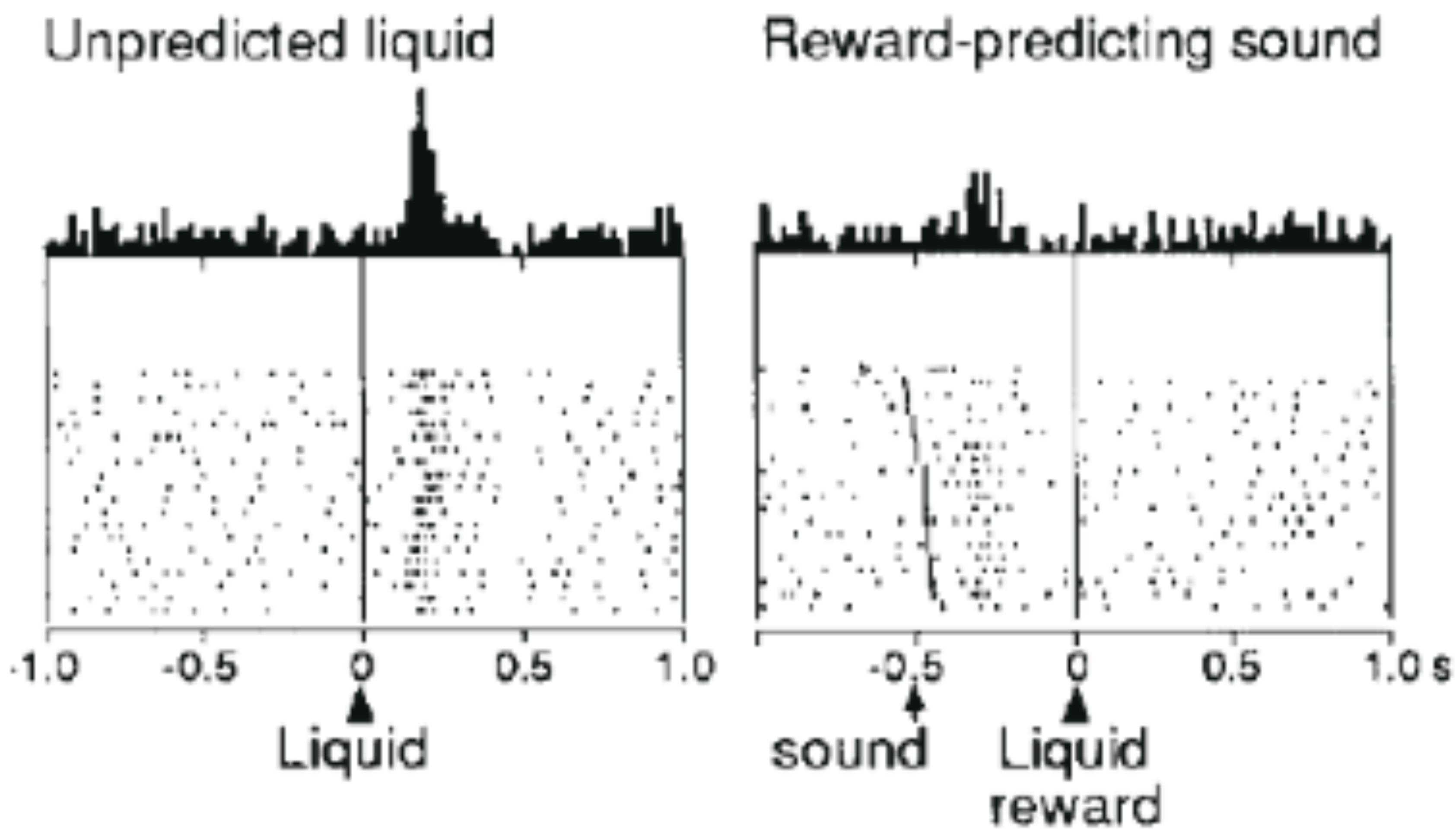
$$\hat{V}(s_t) = r_t + \gamma V(s_{t+1})$$

with the **current estimate** of the value function $V(s_t)$

- ⇨ if $\hat{V}(s_t) > V(s_t)$ than $V(s_t)$ is increased
- ⇨ if $\hat{V}(s_t) < V(s_t)$ than $V(s_t)$ is decreased

Dopamine as TD-error?

Temporal difference error signals can be measured in the brain of monkeys



Monkey brains seem to have it...

Algorithmic Description of TD Learning

Init: $V_0^*(s) \leftarrow 0$

Repeat $t = t + 1$

Observe transition (s_t, a_t, r_t, s_{t+1})

Compute TD error $\delta_t = r_t + \gamma V_t(s_{t+1}) - V_t(s_t)$

Update V-Function $V_{t+1}(s_t) = V_t(s_t) + \alpha\delta_t$

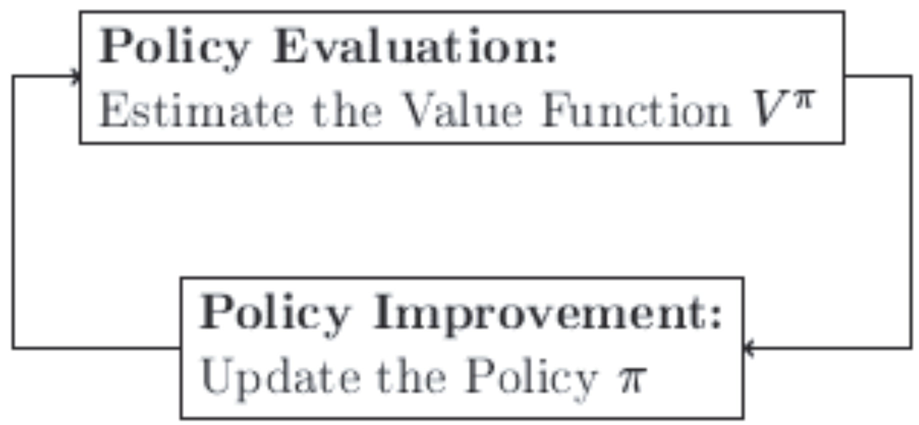
until convergence of V

➡ Used to compute Value function of behavior policy

➡ Sample-based version of policy evaluation

Temporal difference learning for control

So far: Policy evaluation with TD methods



Can **we also do the policy improvement step with samples**?

Yes, but we need to enforce exploration!

Epsilon-Greedy Policy: $$\pi(\boldsymbol{a}|\boldsymbol{s}) = \begin{cases} 1 - \epsilon + \epsilon/|\mathcal{A}|, & \text{if } \boldsymbol{a} = \text{argmax}_{\boldsymbol{a}'} Q^\pi(\boldsymbol{s}, \boldsymbol{a}') \\ \epsilon/|\mathcal{A}, & \text{otherwise} \end{cases}$$

Soft-Max Policy: $$\pi(\boldsymbol{a}|\boldsymbol{s}) = \frac{\exp(\beta Q(\boldsymbol{s}, \boldsymbol{a}))}{\sum_{\boldsymbol{a}'} \exp(\beta Q(\boldsymbol{s}, \boldsymbol{a}'))}$$

➡ **Do not always take greedy action**

Temporal difference learning for control

Update equations for learning the Q-function $Q(s, a)$

$$Q_{t+1}(s_t, a_t) = Q_t(s_t, a_t) + \alpha\delta_t, \quad \delta_t = r_t + \gamma Q_t(s_{t+1}, a_?) - Q_t(s_t, a_t)$$

Two different methods to estimate $a_?$

Q-learning: $a_? = \mathrm{argmax}_a Q_t(s_{t+1}, a)$

Estimates Q-function of optimal policy

Off-policy samples: $a_? \neq a_{t+1}$

SARSA: $a_? = a_{t+1}$ **,** where $a_{t+1} \sim \pi(a|s_{t+1})$

Estimates Q-function of exploration policy

On-policy samples

Note: The policy for generating the actions depends on the Q-function ➡ non-stationary policy

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Final Remarks

Approximating the Value Function

In the continuous case, we need to approximate the V-function (except for LQR)

Lets keep it simple, we use **a linear model** to represent the V-function

$$V^{\pi}(\boldsymbol{s}) \approx V_{\boldsymbol{\omega}}(\boldsymbol{s}) = \boldsymbol{\phi}^T(\boldsymbol{s})\boldsymbol{\omega}$$

How can we find the parameters ω ?

➡ Again with Temporal Difference Learning

TD-learning with Function Approximation

Derivation:

Use the **recursive definition of V-function**:

$$\mathrm{MSE}(\boldsymbol{\omega}) \approx \mathrm{MSE}_{\mathrm{BS}}(\boldsymbol{\omega}) = 1/N \sum_{i=1}^{N} \left(\hat{V}^{\pi}(\boldsymbol{s}_i) - V_{\boldsymbol{\omega}}(\boldsymbol{s}_i) \right)^2$$

with $\hat{V}^{\pi}(\boldsymbol{s}) = \mathbb{E}_{\pi}\left[r(\boldsymbol{s}, \boldsymbol{a}) + \mathbb{E}_{\mathcal{P}}\left[V_{\boldsymbol{\omega}_{\mathrm{old}}}(\boldsymbol{s}') | \boldsymbol{s}, \boldsymbol{a} \right] \right]$

⇨ **Bootstrapping (BS):** Use the old approximation to get the target values for a new approximation

How can we **minimize** this function ?

Lets use **stochastic gradient descent**

Refresher: Stochastic Gradient Descent

Consider an **expected error function,**

$$E_{\boldsymbol{\omega}} = \mathbb{E}_p[e_{\boldsymbol{\omega}}(x)] \approx 1/N \sum_{i=1}^{N} e_{\boldsymbol{\omega}}(x_i), \quad x_i \sim p(x)$$

We can find a local minimum of E by **Gradient descent:**

$$\boldsymbol{\omega}_{k+1} = \boldsymbol{\omega}_k - \alpha_k \frac{dE_{\boldsymbol{\omega}}}{d\boldsymbol{\omega}} = \boldsymbol{\omega}_k - \alpha_k \sum_{i=1}^{N} \frac{de_{\boldsymbol{\omega}}(x_i)}{d\boldsymbol{\omega}}$$

Stochastic Gradient Descent does the gradient update already after a **single sample**

$$\boldsymbol{\omega}_{k+1} = \boldsymbol{\omega}_k - \alpha_k \frac{de_{\boldsymbol{\omega}}(x_k)}{d\boldsymbol{\omega}}$$

Converges under the stochastic approximation conditions

$$\sum_{k=1}^{\infty} \alpha_k = \infty, \quad \sum_{k=1}^{\infty} \alpha_k^2 < \infty$$

Temporal difference learning

Stochastic gradient descent on our error function MSE_{BS}

$$\mathrm{MSE}_{\mathrm{BS,t}}(\boldsymbol{\omega}) = 1/N \sum_{i=1}^{N} \left(\hat{V}(\boldsymbol{s}_t) - V_{\boldsymbol{\omega}}(\boldsymbol{s}_i)\right)^2$$

$$= 1/N \sum_{i=1}^{N} \left(r_i + \gamma V_{\boldsymbol{\omega}_t}(\boldsymbol{s}_i') - V_{\boldsymbol{\omega}}(\boldsymbol{s}_i)\right)^2$$

Update rule (for current time step t, $V_{\boldsymbol{\omega}}(\boldsymbol{s}) = \boldsymbol{\phi}^T(\boldsymbol{s})\boldsymbol{\omega}$)

$$\boldsymbol{\omega}_{t+1} = \boldsymbol{\omega}_t + \alpha_t \left. \frac{d\mathrm{MSE}_{\mathrm{BS}}}{d\boldsymbol{\omega}} \right|_{\boldsymbol{\omega}=\boldsymbol{\omega}_t}$$

$$\boldsymbol{\omega}_{t+1} = \boldsymbol{\omega}_t + \alpha \Big(r(\boldsymbol{s}_t, \boldsymbol{a}_t) + \gamma V_{\boldsymbol{\omega}_t}(\boldsymbol{s}_{t+1}) - V_{\boldsymbol{\omega}_t}(\boldsymbol{s}_t) \Big) \boldsymbol{\phi}^T(\boldsymbol{s}_t)$$

$$= \boldsymbol{\omega}_t + \alpha \delta_t \boldsymbol{\phi}^T(\boldsymbol{s}_t)$$

with $\delta_t = r(\boldsymbol{s}_t, \boldsymbol{a}_t) + \gamma V_{\boldsymbol{\omega}_t}(\boldsymbol{s}_{t+1}) - V_{\boldsymbol{\omega}_t}(\boldsymbol{s}_t)$

Temporal difference learning

TD with function approximation

$$\boldsymbol{\omega}_t = \boldsymbol{\omega}_t + \alpha \delta_t \boldsymbol{\phi}^T(\boldsymbol{s}_t)$$

Difference to discrete algorithm:

- TD-error is correlated with the feature vector
- Equivalent if tabular feature coding is used, i.e., $\phi(s_i) = \boldsymbol{e}_i$

Similar update rules can be obtained for SARSA and Q-learning

$$\boldsymbol{\omega}_{t+1} = \boldsymbol{\omega}_t + \alpha \Big(r(\boldsymbol{s}_t, \boldsymbol{a}_t) + \gamma Q_{\boldsymbol{\omega}_t}(\boldsymbol{s}_{t+1}, \boldsymbol{a}_?) - Q_{\boldsymbol{\omega}_t}(\boldsymbol{s}_t, \boldsymbol{a}_t) \Big) \boldsymbol{\phi}^T(\boldsymbol{s}_t, \boldsymbol{a}_t)$$

where $Q_{\boldsymbol{\omega}}(\boldsymbol{s}, \boldsymbol{a}) \approx \boldsymbol{\phi}^T(\boldsymbol{s}, \boldsymbol{a}) \boldsymbol{\omega}$

Temporal difference learning

Some remarks on temporal difference learning:

- Its **not a proper** stochastic gradient descent!!
- **Why?** Target values $\hat{V}^{\pi}(\boldsymbol{s})$ **change after each parameter update!**

  We ignore the fact that $\hat{V}^{\pi}(\boldsymbol{s})$ also depends on $\boldsymbol{\omega}$
- **Side note:** This „ignorance“ actually introduces a bias in our optimization, such that we are optimizing a different objective than the *MSE*
- In certain cases, we also get **divergence** (e.g. off-policy samples)
- TD-learning is very fast in terms of computation time O(#features), but not data-efficient ⇨ **each sample is just used once!**

Dann, Neumann, Peters: *Policy Evaluation with Temporal Differences: A survey and comparison*, JMLR, in press

Sucessful examples

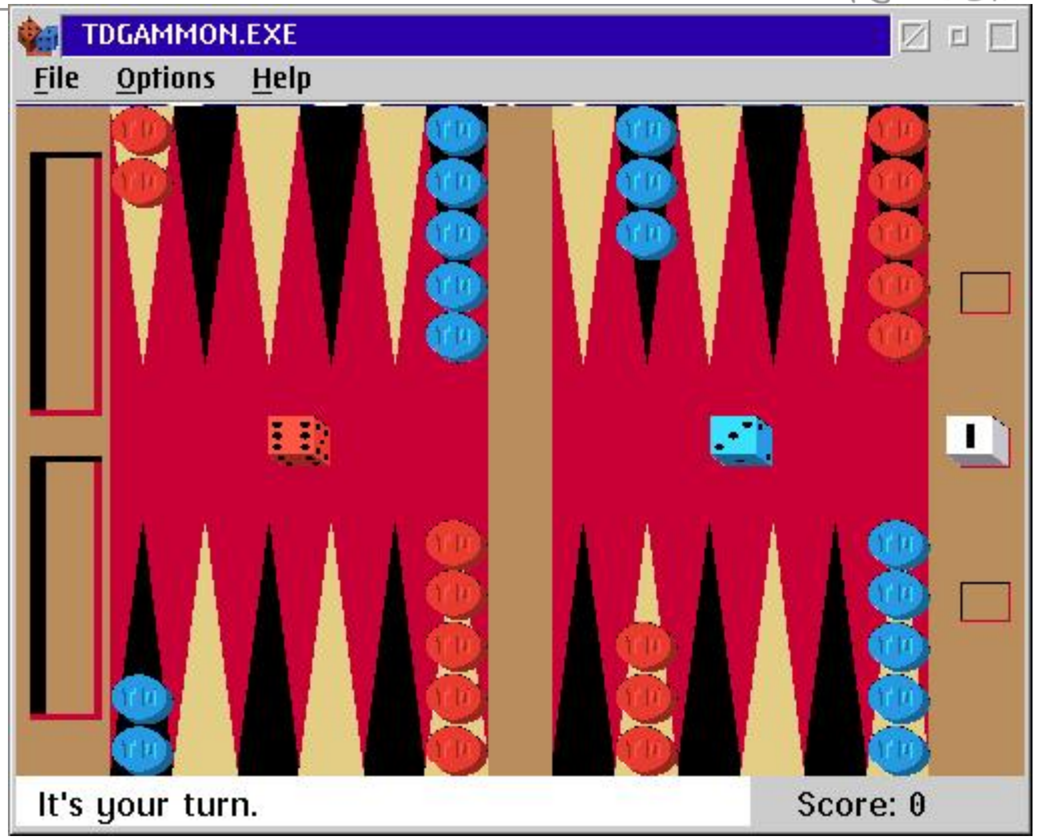
Linear function approximation

Tetris, Go

Non-linear function approximation

TD Gammon (Worldchampion level)

Atari Games (learning from raw pixel input)



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Final Remarks

Batch-Mode Reinforcement Learning

Online methods are typically **data-inefficient** as they use each data point only once

$$D = \left\{ \boldsymbol{s}_i, \boldsymbol{a}_i, r_i, \boldsymbol{s}'_i \right\}_{i=1\ldots N}$$

Can we **re-use the whole „batch“** of data to **increase data-efficiency?**

- **Least-Squares Temporal Difference (LSTD) Learning**
- **Fitted Q-Iteration**

⇨ Computationally **much more expensive** then TD-learning!

Least-Squares Temporal Difference (LSTD)

Lets **minimize the bootstrapped *MSE*** objective (*MSE*$_{BS}$)

$$\mathrm{MSE_{BS}} = 1/N \sum_{i=1}^{N} \left(r(\boldsymbol{s}_i, \boldsymbol{a}_i) + \gamma V_{\boldsymbol{\omega}_{\mathrm{old}}}(\boldsymbol{s}_i') - V_{\boldsymbol{\omega}}(\boldsymbol{s}_i) \right)^2$$

$$= 1/N \sum_{i=1}^{N} \left(r(\boldsymbol{s}_i, \boldsymbol{a}_i) + \gamma \boldsymbol{\phi}^T(\boldsymbol{s}_i') \boldsymbol{\omega}_{\mathrm{old}} - \boldsymbol{\phi}^T(\boldsymbol{s}_i) \boldsymbol{\omega} \right)^2$$

Least-Squares Solution:

$$\boldsymbol{\omega} = (\boldsymbol{\Phi}^T \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^T (\boldsymbol{R} + \gamma \boldsymbol{\Phi}' \boldsymbol{\omega}_{\mathrm{old}})$$

with $\boldsymbol{\Phi} = [\boldsymbol{\phi}(\boldsymbol{s}_1), \boldsymbol{\phi}(\boldsymbol{s}_2), \dots, \boldsymbol{\phi}(\boldsymbol{s}_N)]^T$

$$\boldsymbol{\Phi}' = [\boldsymbol{\phi}(\boldsymbol{s}_1'), \boldsymbol{\phi}(\boldsymbol{s}_2'), \dots, \boldsymbol{\phi}(\boldsymbol{s}_N')]^T$$

Least-Squares Temporal Difference (LSTD)

Least-Squares Solution:

$$\boldsymbol{\omega} = (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T(\boldsymbol{R} + \gamma\boldsymbol{\Phi}'\boldsymbol{\omega}_{\text{old}})$$

Fixed Point: In case of convergence, we want to have $\boldsymbol{\omega}_{\text{old}} = \boldsymbol{\omega}$

$$\begin{aligned}
\boldsymbol{\omega} &= (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T(\boldsymbol{R} + \gamma\boldsymbol{\Phi}'\boldsymbol{\omega}) \\
\left(\boldsymbol{I} - \gamma(\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\boldsymbol{\Phi}'\right)\boldsymbol{\omega} &= (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\boldsymbol{R} \\
(\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\left(\boldsymbol{\Phi} - \gamma\boldsymbol{\Phi}'\right)\boldsymbol{\omega} &= (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\boldsymbol{R} \\
\boldsymbol{\Phi}^T\left(\boldsymbol{\Phi} - \gamma\boldsymbol{\Phi}'\right)\boldsymbol{\omega} &= \boldsymbol{\Phi}^T\boldsymbol{R}
\end{aligned}$$

$$\boldsymbol{\omega} = \left(\boldsymbol{\Phi}^T(\boldsymbol{\Phi} - \gamma\boldsymbol{\Phi}')\right)^{-1}\boldsymbol{\Phi}^T\boldsymbol{R}$$

Least-Squares Temporal Difference (LSTD)

LSTD solution:

$$\boldsymbol{\omega} = \left(\boldsymbol{\Phi}^T(\boldsymbol{\Phi} - \gamma\boldsymbol{\Phi}')\right)^{-1}\boldsymbol{\Phi}^T\boldsymbol{R}$$

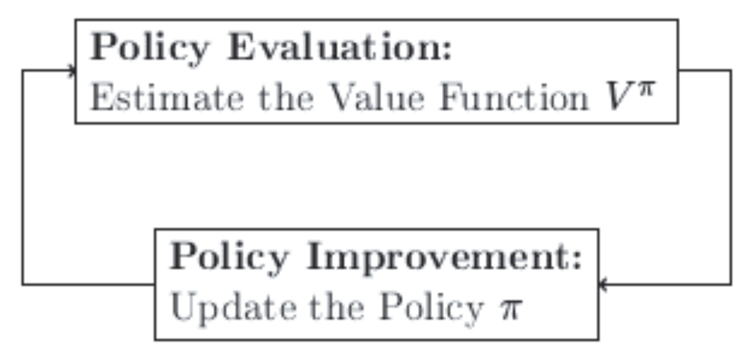
Same solution as convergence point of **TD-learning**

One shot! **No iterations** necessary for policy evaluation

LSQ: Adaptation for learning the Q-function

$$\boldsymbol{\Phi} = [\phi(\boldsymbol{s}_1, \boldsymbol{a}_1), \phi(\boldsymbol{s}_2, \boldsymbol{a}_2), \ldots, \phi(\boldsymbol{s}_N, \boldsymbol{a}_N)]^T$$

$$\boldsymbol{\Phi}' = [\phi(\boldsymbol{s}_2, \boldsymbol{a}_2), \phi(\boldsymbol{s}_3, \boldsymbol{a}_3), \ldots, \phi(\boldsymbol{s}_{N+1}, \boldsymbol{a}_{N+1})]^T$$



⇨ Used for **Least-Squares Policy Iteration (LSPI)**

Lagoudakis and Parr, ***Least-Squares Policy Iteration,*** *JMLR*

Learning to Ride a Bicycle

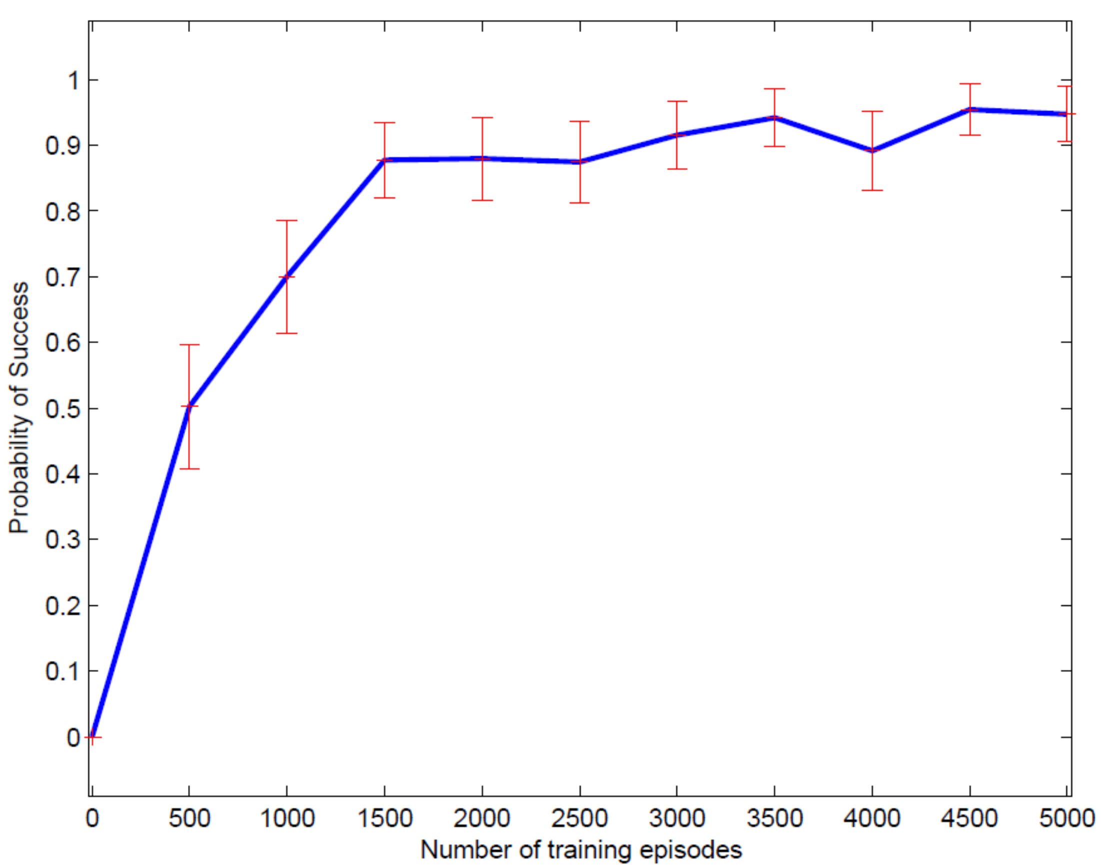
State space: $s = [\theta, \dot{\theta}, \omega, \dot{\omega}, \ddot{\omega}, \psi]$

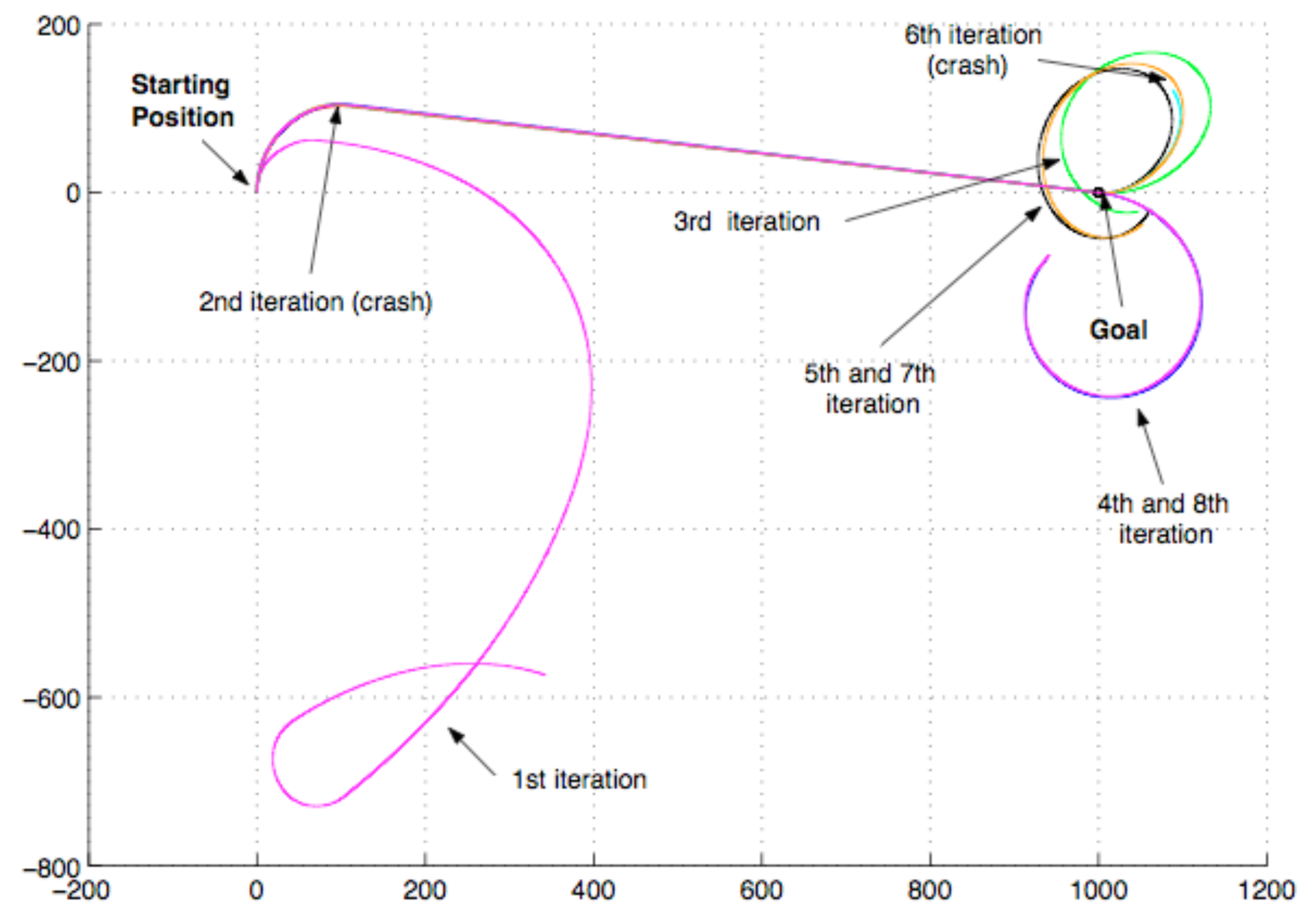
θ angle of handlebar, ω vertical angle of bike, ψ angle to goal

Action space: 5 discrete actions (torque applied to handle, displacement of rider)

Feature space: 20 basis functions…

$$(\ 1,\ \omega,\ \dot{\omega},\ \omega^2,\ \dot{\omega}^2, \omega\dot{\omega},\ \theta,\ \dot{\theta},\ \theta^2,\ \dot{\theta}^2,\ \theta\dot{\theta},\ \omega\theta,\ \omega\theta^2,\ \omega^2\theta,\ \psi,\ \psi^2,\ \psi\theta,\ \bar{\psi},\ \bar{\psi}^2,\ \bar{\psi}\theta\ )^{\mathsf{T}}$$





Fitted Q-iteration

In Batch-Mode RL it is also much easier to use **non-linear function approximators**

- Many of them **only exists in the batch setup**, e.g. regression trees
- **No catastrophic forgetting**, e.g., for neural networks.
- Strong divergence problems, fixed for Neural Networks by ensuring that there is a goal state where the Q-Function value is always zero (see Lange et al. below).

Fitted Q-iteration uses non-linear function approximators for **approximate value iteration.**

Ernst, Geurts and Wehenkel, *Tree-Based Batch Mode Reinforcement Learning, JMLR 2005*
Lange, Gabel and Riedmiller. *Batch Reinforcement Learning*, *Reinforcement Learning: State of the Art*

Fitted Q-iteration

Given: Dataset $D = \left\{ \boldsymbol{s}_i, \boldsymbol{a}_i, r_i, \boldsymbol{s}'_i \right\}_{i=1\ldots N}$

Algorithm:

Initialize $Q^{[0]}(\boldsymbol{s}, \boldsymbol{a}) = 0$, input data: $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{s}_1^T & \boldsymbol{a}_1^T \\ \vdots & \\ \boldsymbol{s}_N^T & \boldsymbol{a}_N^T \end{bmatrix}$

for k = 1 to L

Generate target values: $\tilde{q}_i^{[k]} = r_i + \gamma \max_{\boldsymbol{a}'} Q^{[k-1]}(\boldsymbol{s}'_i, \boldsymbol{a}')$

Learn new Q-function: $Q^{[k]}(\boldsymbol{s}, \boldsymbol{a}) \leftarrow \mathrm{Regress}(\boldsymbol{X}, \tilde{\boldsymbol{q}}^{[k]})$

end

➡ Like Value-Iteration, but we use supervised learning methods to approximate the Q-function at each iteration k

Fitted Q-iteration

Some Remarks:

- Regression does the **expectation** for us

$$Q^{[k]}(\boldsymbol{s}, \boldsymbol{a}) \approx \mathbb{E}_{\mathcal{P}}\left[r(\boldsymbol{s}, \boldsymbol{a}) + \gamma \max_{\boldsymbol{a}'} Q^{[k-1]}(\boldsymbol{s}', \boldsymbol{a}')\right]$$

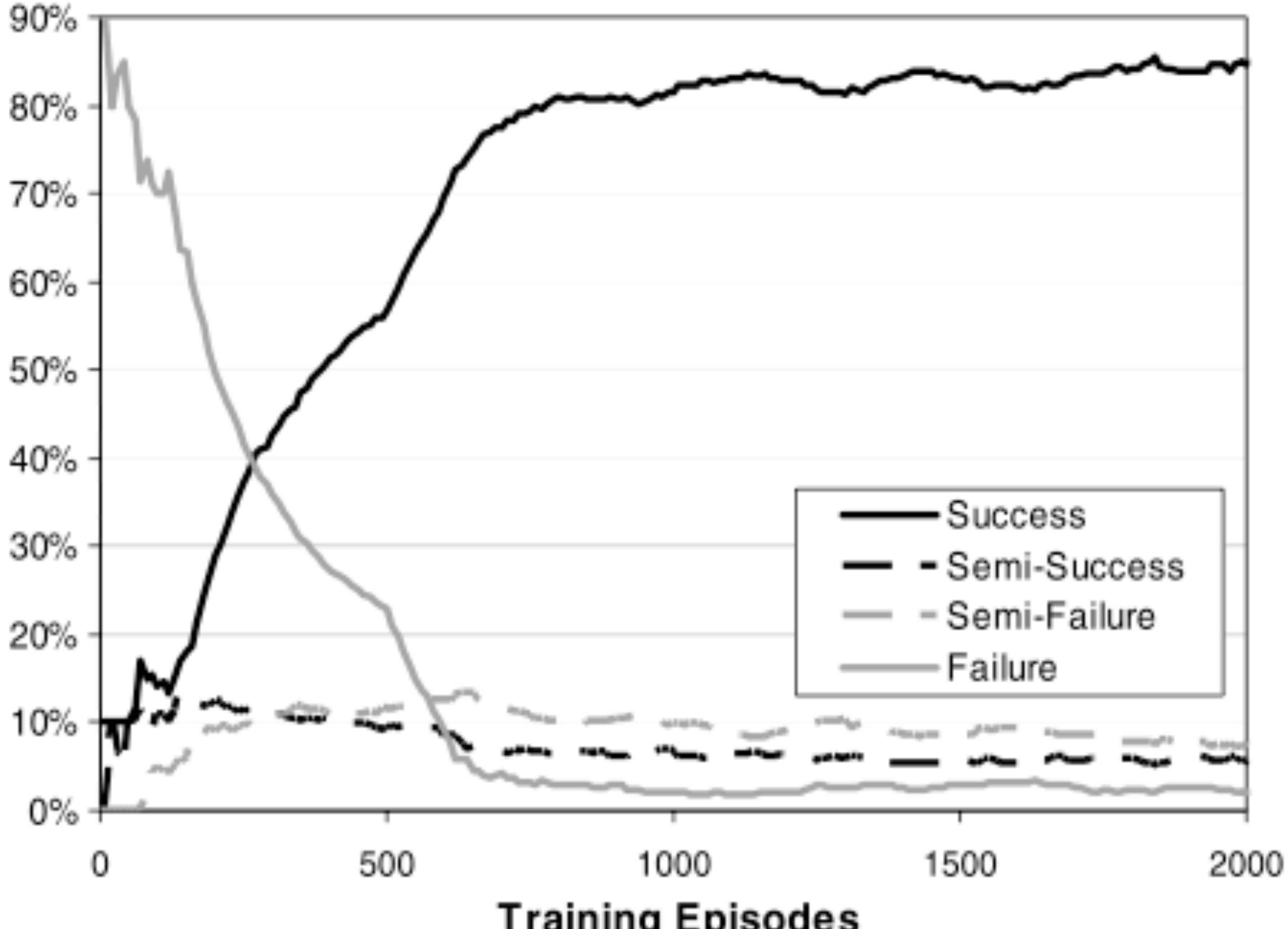
- The max operator is still **hard to solve for continous action spaces**

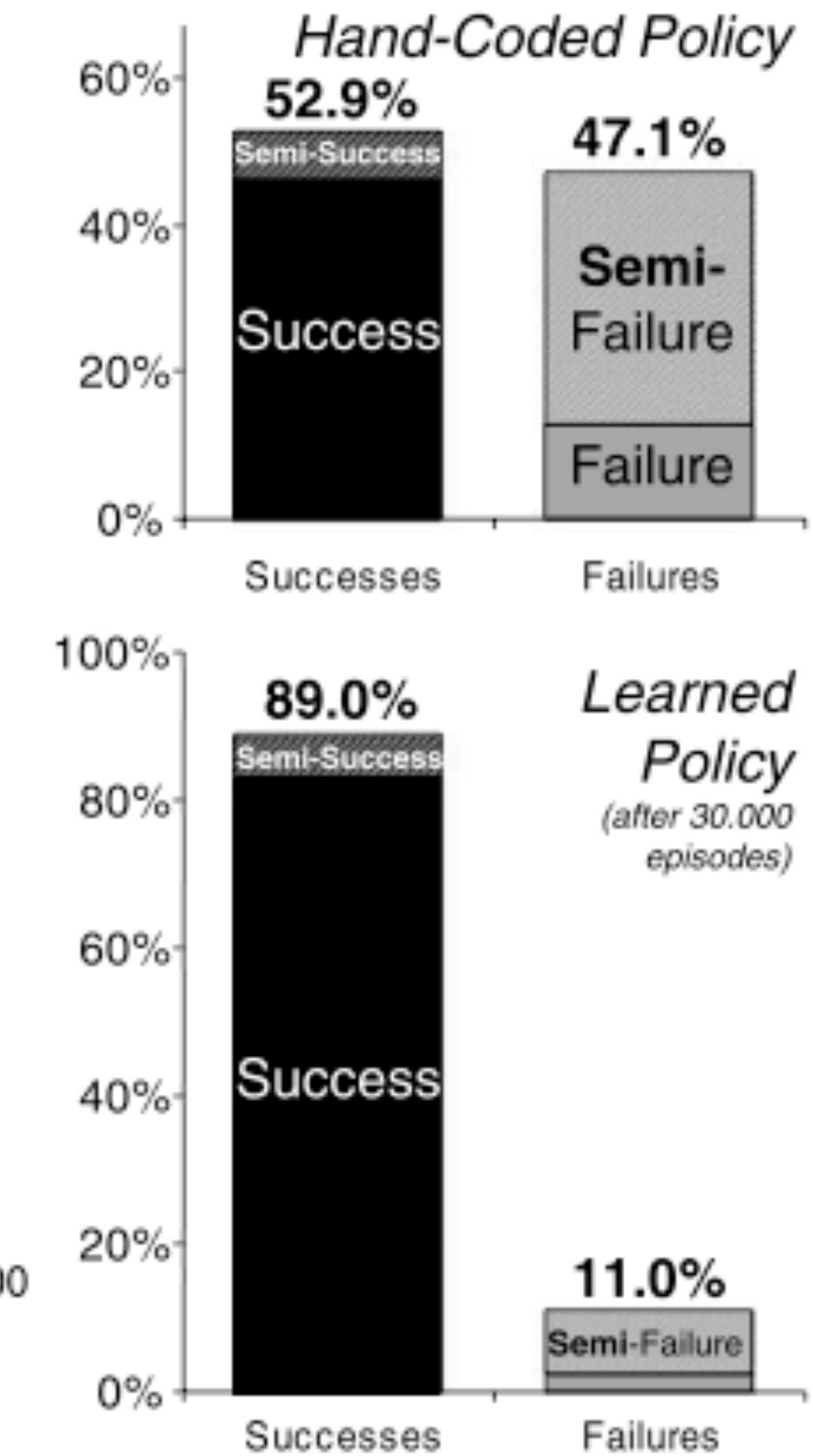
For continuous actions, see: Neumann and Peters, *Fitted Q-iteration by Advantage weighted regression*, NIPS, 2008

Case Study I: Learning Defense

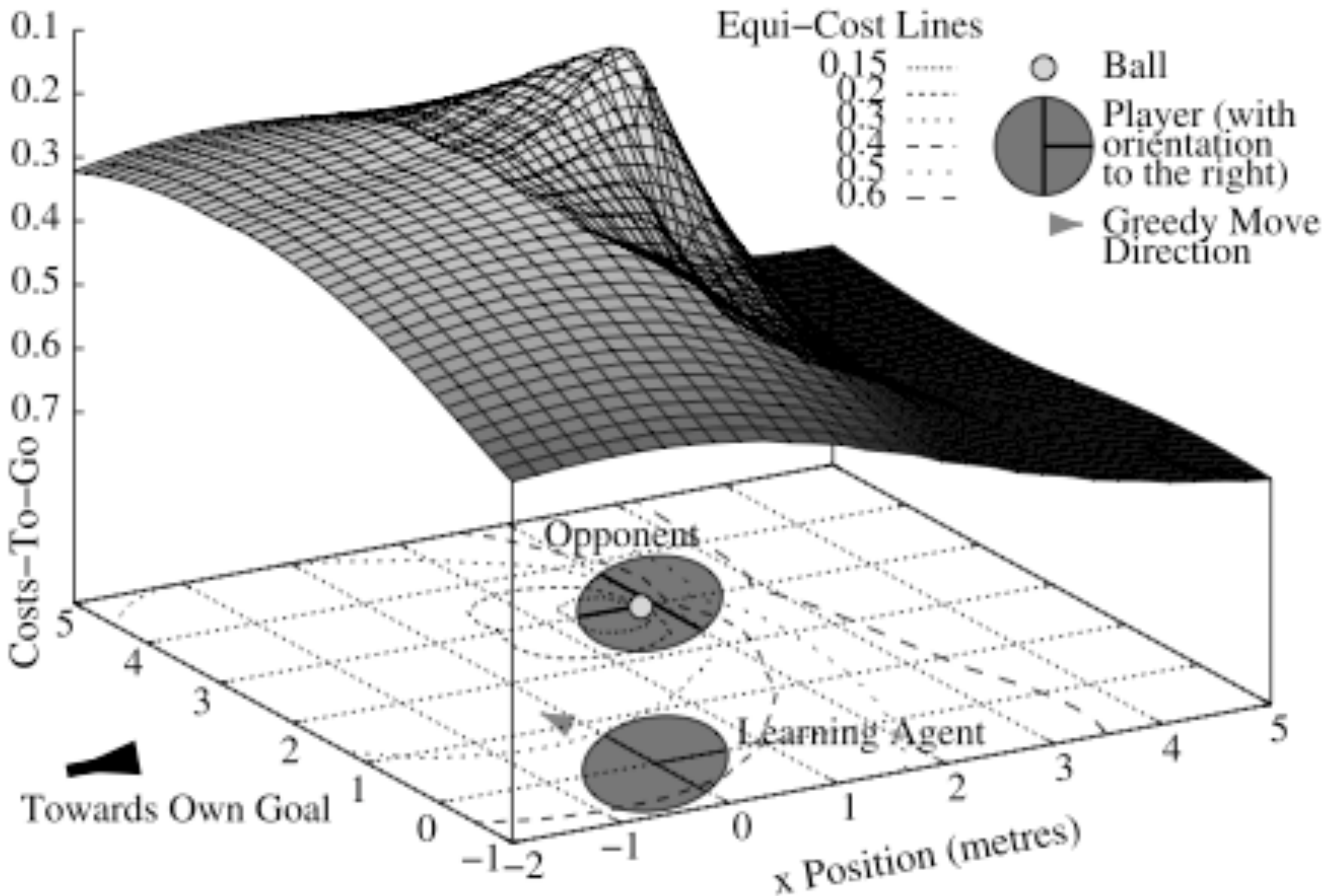
Within the RL framework, we model the ADB learning task as a terminal state problem with both terminal goal S^+ and failure states S^-. Intermediate steps are punished by constant costs of $c = 0.05$, whereas $J(s) = 0.0$ for $s \in S^+$ and $J(s) = 1.0$ for $s \in S^-$ by definition (cf. Eq. 8).

Success

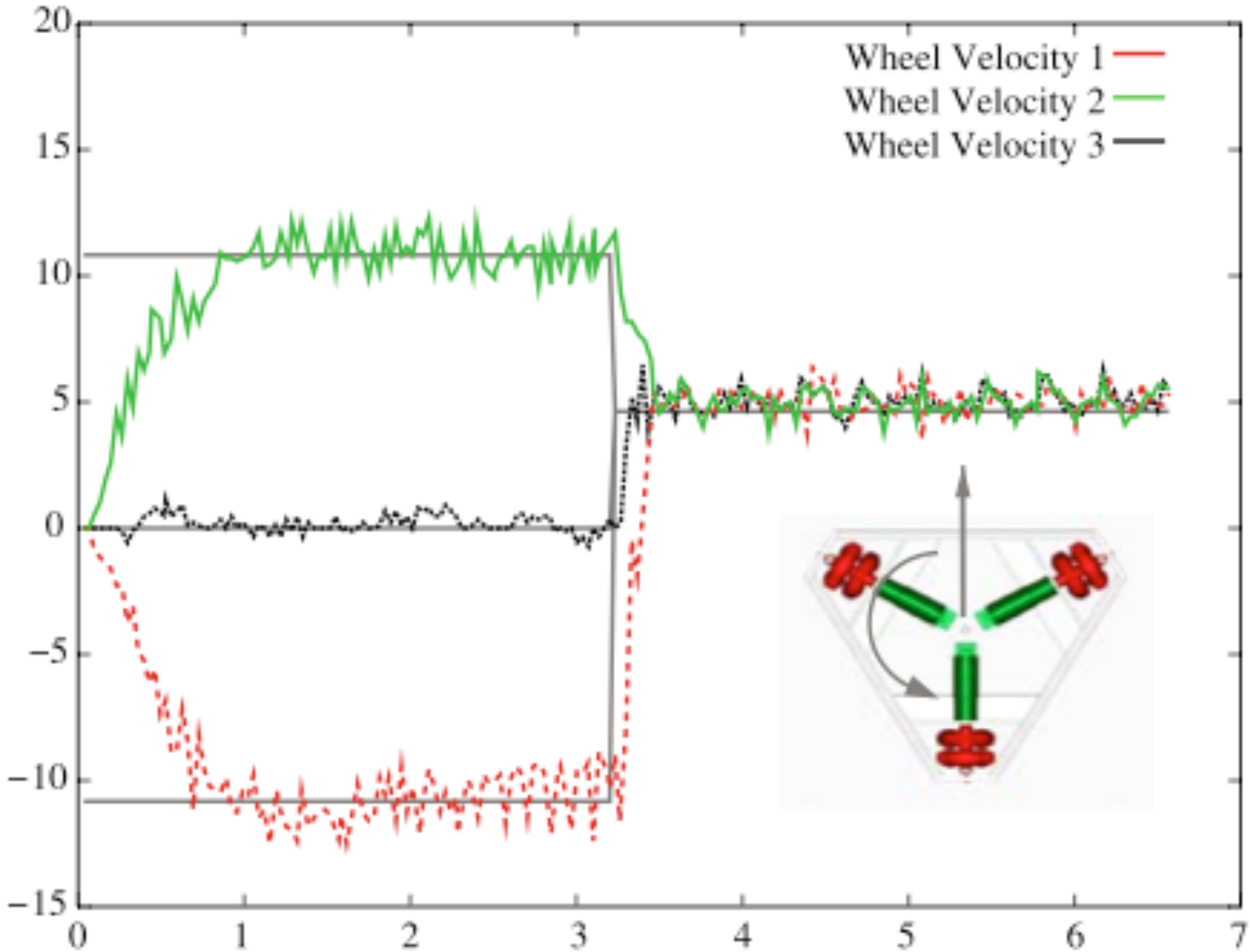

90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
500
1000
1500
2000
Training Episodes
Success
Semi-Success
Semi-Failure
Failure


Hand-Coded Policy
60%
40%
20%
0%
52.9%
Semi-Success
Success
47.1%
Semi-
Failure
Failure
Successes
Failures
Learned Policy
(after 30.000 episodes)
100%
80%
60%
40%
20%
0%
89.0%
Semi-Success
Success
11.0%
Semi-Failure
Successes
Failures

Dueling Behavior

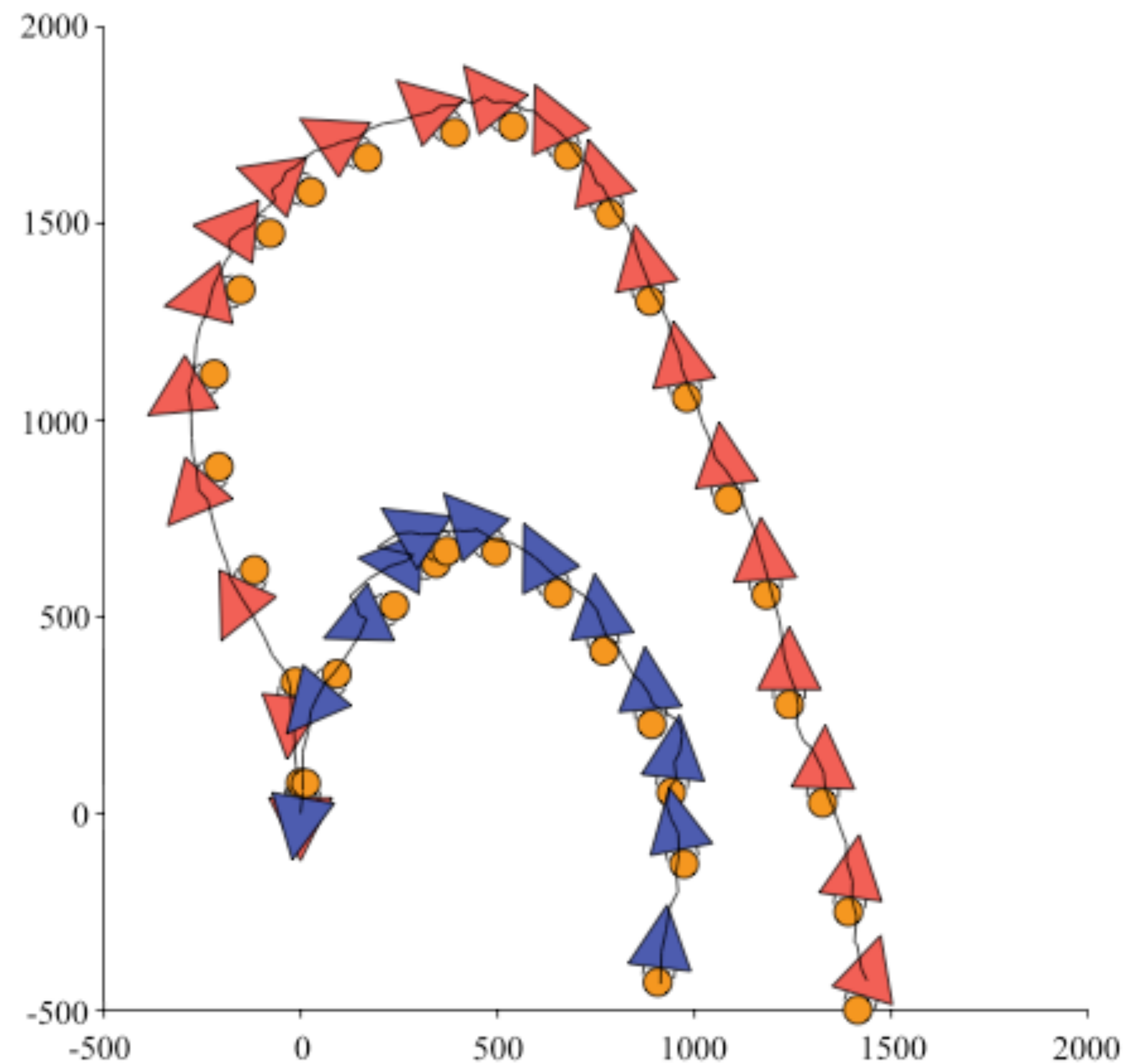


Case Study II: Learning Motor Speeds



$$c(s, a, s') = c(s) = \begin{cases} 0 & \text{if } |\dot{\omega}_d - \dot{\omega}| < \delta, \\ 0.01 & \text{else.} \end{cases}$$

Case Study III: Learning to Dribble



$$Q^{target}(s,a) := \begin{cases} 1.0, & \text{if } s' \in S^-, \\ 0.01, & \text{if } s' \in S^+, \\ 0.01 + \min_b \tilde{Q}(s',b), & \text{else} \end{cases}$$

Value Function Methods

- ... have been the driving reinforcement learning approach in the 1990s.
- You can do loads of cool things with them: Learn Chess at professional level, learn **Backgammon and Checkers at Grandmaster-Level** ... and winning the **Robot Soccer Cup** with a minimum of man power.

So, why are they not always the method of choice?

- You need to fill-up you state-action space up with sufficient samples.
- Another curse of dimensionality with an exponential explosion.
- Errors in the Value function approximation might have a catastrophic effect on the policy, can be very hard to control
- However, it scales better as we only need samples at relevant locations.