Underactuated Trajectory-Tracking Control for Long-Exposure Photography

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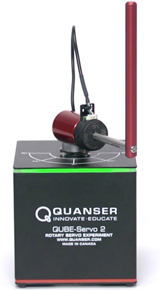

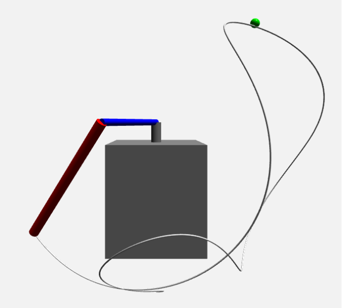

Motivation

Underactuated systems cannot follow arbitrary trajectories.

- How to find feasible trajectories?
- Can these trajectories be tracked?

- Only sparse research available

Problem Statement

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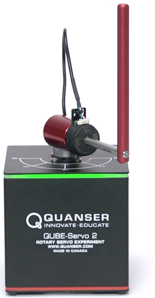
- **Control** the Quanser Qube
  $\rightarrow$ Rotary Inverted Pendulum



- Do **light-painting** with the tip of the pendulum using long exposure photography
- e.g. Draw letters like ’I’, ’A’ and ’S’

Approach

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1) Planning

- Offline trajectory planning
- Direct collocation & RBF-based objective

2) Trajectory Tracking

- Track the planned trajectory
- LQR feedback control

3) Long Exposure Photography

System Identification

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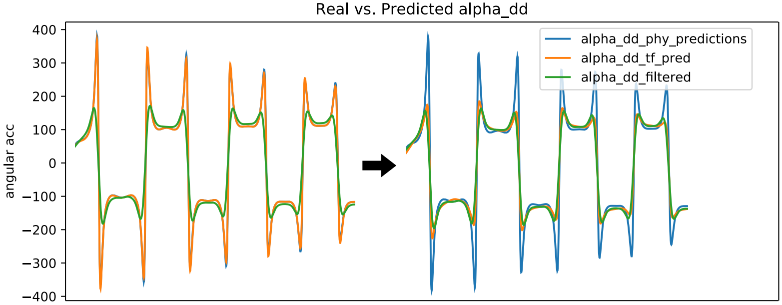
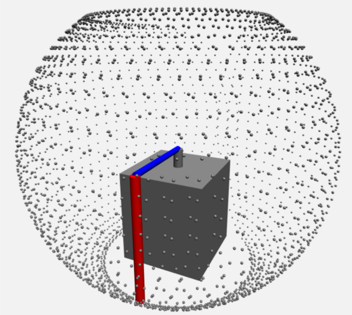
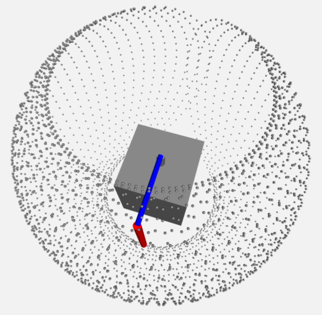
- **Approach:** Gradient descent on the dynamic model
- Tensorflow graph of the dynamics
- Parameters initialized with the values given by the manufacturer



Task Space Analysis

- Surface of a sphere with cut off bottom and top





Point Projection between Camera Space and Task Space

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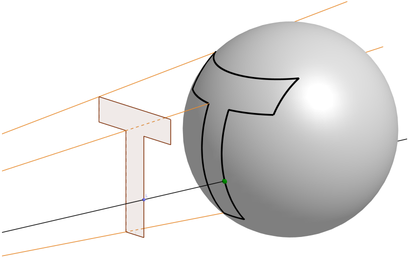
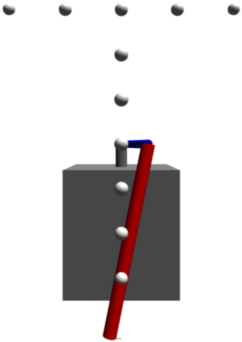
Continous **projection** from camera to task space



Letter **discretization**



Trajectory Planning

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Goal

- Generate smooth trajectorthrough the via points

Approach

- Direct Collocation
- RBF based objective function
- Additional constraints

Objective Function

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$$J = \sum_{k=0}^{N-1} \left(\sum_{j=0}^{M-1} e^{-(t_j - k)^2} d\left(\mathbf{x}_k, \mathbf{x}_j^{\text{target}}\right) + \beta u_k^2 \right) + \alpha \left(t_{N-1} - t_0\right)$$

N : Number of time steps

M : Number of target points

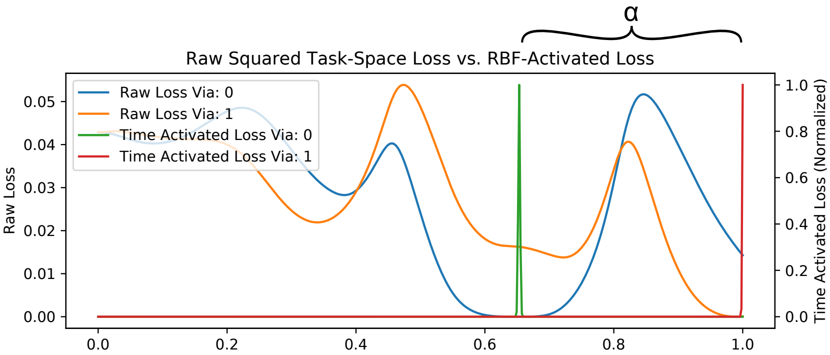
$d(\cdot)$: Distance metric $\rightarrow$ we use squared task space difference

α : Trade-off factor for the time differences

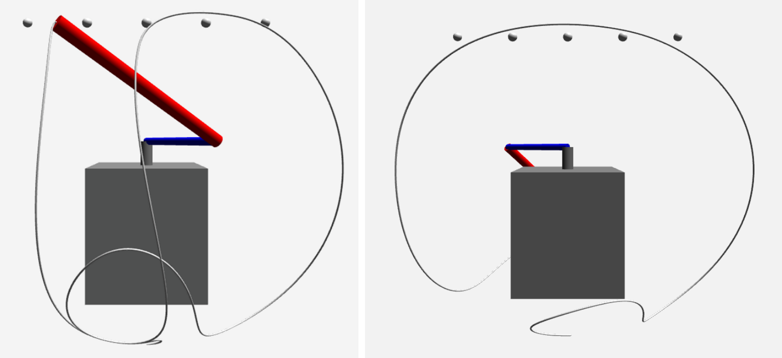
β : Trade-off factor for punishing high control values

$u_k, \mathbf{x}_k, t_j$: Optimization variables

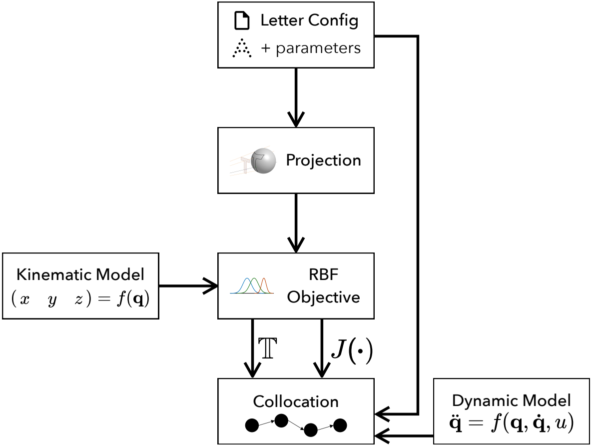
RBF Based Objective



Trading Off Time Differences



Trajectory Planning Pipeline



Practical Tricks and Challenges

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Practical Tricks

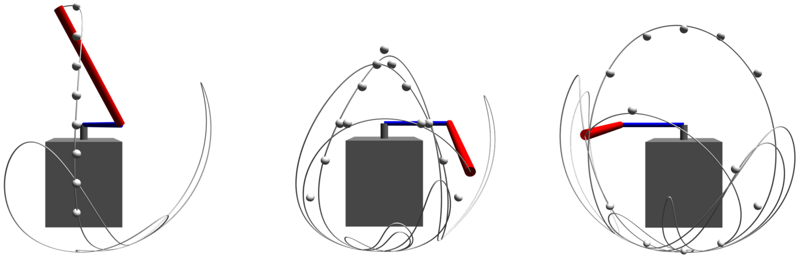
- Constrain the order of via points: $t_j < t_{j+1}$
- Constrain the duration between activations of subsequent points
- Constraint control signals $\rightarrow$ improves controllability

Challenges

- Complex objective function
- Hyperparameters: trade-off factor, target points order, trajectory duration, ...

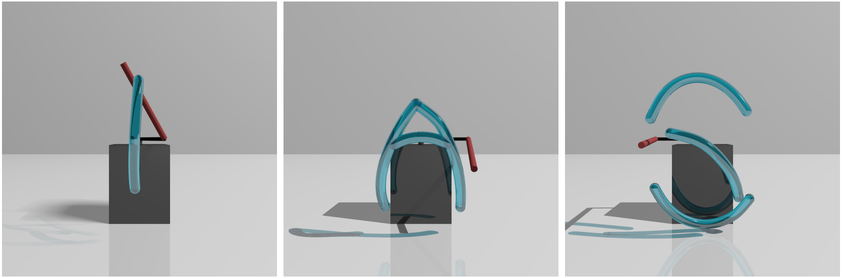
Finding good solution for complex letters requires a lot of hand-tuning

Results

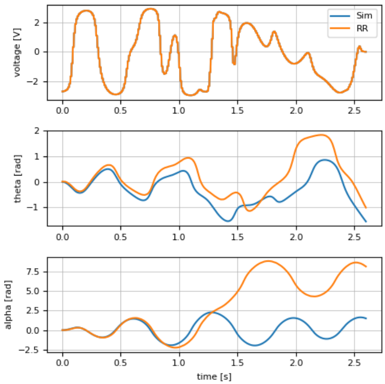


Results

- Time-based activated traces
- External render tool



Open Loop Control



- Open Loop not possible
- **Idea**: Use the optimal feedback control of LQR in context of trajectory tracking

Linear Quadratic Regulator

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- Linearize system dynamics around desired trajectory $\{(u_k^d, \mathbf{x}_k^d)\}_{k=0,\dots,N}$
- Use finite-horizon **Linear Quadratic Regulator** (LQR)

Objective

$$J = (\mathbf{x}_N - \mathbf{x}_N^d)^T \mathbf{Q} (\mathbf{x}_N - \mathbf{x}_N^d) + \sum_{t=0}^{N-1} (\mathbf{x}_t - \mathbf{x}_t^d)^T \mathbf{Q} (\mathbf{x}_t - \mathbf{x}_t^d) + (\mathbf{u}_t - \mathbf{u}_t^d)^T \mathbf{R} (\mathbf{u}_t - \mathbf{u}_t^d)$$

Feedback Control Law

$$\mathbf{u}_t = \mathbf{u}_t^d - \mathbf{K}_t (\mathbf{x}_t - \mathbf{x}_t^d)$$

Trajectory Tracking

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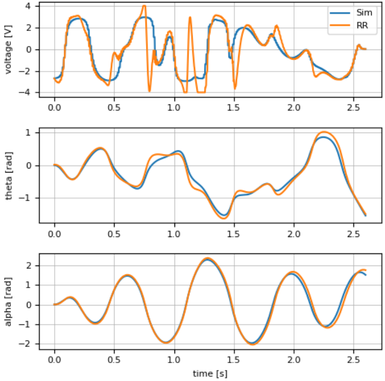

- Same desired trajectory
- LQR feedback control

Challenges

- No guarantee that trajectory is trackable
- Weight matrices **Q** and **R** need to be hand tuned

Long Exposure Photography

$$\mathbb{P} = [0, 255]^3, \quad \mathbb{F} = \mathbb{P}^{H \times W}$$
$$\mathbf{V} = \begin{bmatrix} \mathbf{F}_1 & \cdots & \mathbf{F}_T \end{bmatrix} \quad, \mathbf{F}_t \in \mathbb{F}$$
$$\mathbf{L}_{ij} = \max_t (\mathbf{F}_t)_{ij} \quad, \forall i \in [0, H], j \in [0, B]$$

- Maximum value for each pixel over time
- Approximates long exposure photo from video

Conclusion & Outlook

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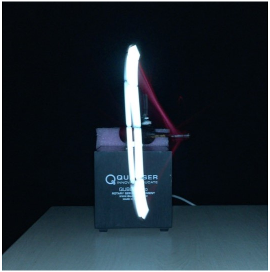
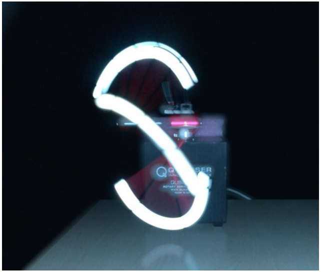
Conclusion

- Planning trajectories can become arbitrary complex for underactuated systems
- Model errors can make trajectory tracking very difficult
- Good tuning of hyperparameters is essential

Outlook

- Automation and optimization of LQR weight parameters





Questions?