2011 IEEE/RSJ International Conference on
Intelligent Robots and Systems
September 25-30, 2011. San Francisco, CA, USA

Improving Operational Space Control of Heavy Manipulators via Open-Loop Compensation

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Abstract— **Operational space control has a number of desirable characteristics but is sensitive to model accuracy. For heavy machines the dynamics are difficult to model due to their friction and dynamic coupling, thus making full compensation imprecise. This work presents an approach in which a simplified model gives partial compensation via an open-loop feedforward input, pre-calculated in forward simulation. In this way, effects that are difficult to compensate for can be partially corrected without causing instability. Since the reference trajectory is known *a priori*, dynamic model parameters are tuned in its neighbourhood, reducing the burden of global modelling. The feasibility and performance of this approach is shown experimentally via improved free motion tracking of an excavator arm. This framework further supports efforts for direct impedance control between bucket tip and soil.**

I. Introduction

Operational space control is a powerful and natural approach for manipulator control but is sensitive to model quality and parameter estimates. If the dynamics are poorly understood, the compensating actions can positively couple causing instability. In part because of this, automation of heavy machinery (and field robots in general) is usually done with robust approaches in joint space, but could profit from operational space control in several respects. Applying operational space control to this class of machinery is not a simple exercise, hence perhaps the sparsity of this of this approach to controlling this class of systems [1].

Consider a hydraulic excavator arm (Fig. 1) as an exemplar of a heavy field/construction manipulator. Some advantages in applying operational space control to this (excavation) domain are as follows: (1) posing the compliance directly in the end-effector frame allows for direct impedance matching between the tool and environment (the bucket and soil in this case), which can lead to a much more efficient use of actuation; (2) more elaborate terrain/excavation models (e.g. those proposed in Refs. [2], [3], [4]) can be directly applied for reaction compensation, since those model forces are usually obtained in task-space; (3) for remote operation, task-level control is intuitive and requires less coordination (while skilled operators handle at maximum three joint simultaneously [5], operational space control of end-effector handles unlimited numbers joints); and, (4) a standard model for subsequent platform/model interoperability. Taken together they can potentially decrease training time and improve the productivity of experienced operators.

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Fig. 1. Field machines such as the ACFR autonomous excavator pictured above are difficult to control directly at task-level due to dependency of the operational space controller on compensation of uncertain dynamics like compliance, friction and excavation reactions.

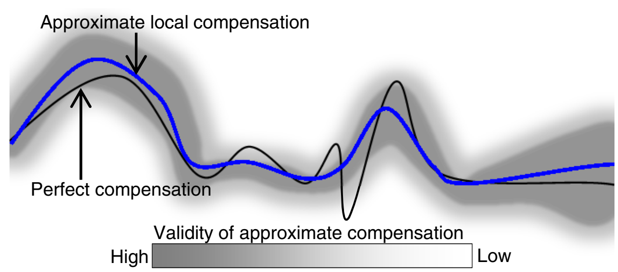


Fig. 2. A conceptual illustration of partial compensation. Modelling fidelity and complexity is relaxed by limiting its prediction range to motions described by the desired trajectory. The model is invalid for other areas (where the shading fades out) and can destabilise a feedback controller. As an open-loop command it is always stable, although the compensation may be biased or incomplete.

The process of dynamic modelling and identification for model-based controllers is an active field in the manipulator control community. In practice, compensation has usually been achieved by using parameter estimation [6], [7] and/or by disturbance observation [8], [9], [10]. Non-parametric modelling based on statistical regression methods [11], [12] has also been used for compensating manipulators, usually with precision mechanical design presenting less friction and/or large redundancy.

Inspired by model predictive methods, this work proposes achieving guaranteed stable compensation by using forward simulation in order to cache the predicted compensation commands for latter use in open-loop operation. Modelling detail is relaxed by using an approximate model for the region around the reference trajectory. As Fig. 2 suggests, the idea is to handle the uncertainty by using a deterministic model to provide conservative partial compensation. This simple model has limited coverage when the current state gets farther from the expected motion, leading to degraded compensation that potentially can cause instability. By taking advantage of the passive stability of heavy machinery (which is often present due to high friction), compensation in the form of open-loop inputs is always stable. If the motion strays far from a prescribed trajectory, the worst result is errant compensation, which results in biased tracking but not in instability.

The paper is organized as follows. In Section II, model compensation issues due to parameter error are discussed. That section also describes the nature of the compliance encountered in heavy hydraulic manipulators. Section III introduces two models for friction compensation and shows that, despite their complexity, a partial model based on viscous friction gives sufficiently motion prediction. Based on this insight, Section IV introduces the central method of this work. Finally, Section V shows the method applied to the experimental platform, followed by conclusions and future work.

II. BACKGROUND

A. Feedback Control

The manipulator dynamics in joint space are given by

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{v}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{dist} \qquad (1)$$

where M is the mass matrix, $\boldsymbol{v}$ is the vector of centrifugal and Coriolis forces, $\boldsymbol{g}$ is the gravity vector, $\boldsymbol{\tau}$ is the vector of input torques and $\boldsymbol{\tau}_{dist}$ the vector of disturbance torques. Since only free motion is considered here, $\boldsymbol{\tau}_{dist}$ reduces to a vector of frictional torque at the joints. Model-based control of manipulators addresses the use of model information in order to compute torques that linearize the dynamics. The compensation commands $\boldsymbol{\tau}_{comp}$ are obtained as

$$\boldsymbol{\tau}_{comp} = \hat{M}(\boldsymbol{q})\boldsymbol{\tau}' + \hat{\boldsymbol{v}}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \hat{\boldsymbol{g}}(\boldsymbol{q}) - \hat{\boldsymbol{\tau}}_{fric} \qquad (2)$$

where the terms on the right side represent the rigid body dynamics and friction estimates (indicated by ($\hat{}$)), and $\boldsymbol{\tau}' = \ddot{\boldsymbol{q}}$ is the vector of input torques for unitary-mass dynamics. If the estimates are perfect, $\boldsymbol{\tau}_{comp}$ completely linearises the dynamics of the manipulator, making it possible to use a linear control law in the outer feedback loop. On the other hand, estimation error appears as disturbances in $\boldsymbol{\tau}_{comp}$. Experimental results show that incorrect estimation of $\hat{\boldsymbol{\tau}}_{fric}$ can lead to instability.

A PD controller in operational space is used for each of the positions of the end-effector (x, y, θ), where gains are specified in terms of the natural frequency f_n (Hz) and damping ζ of the proportional spring for the unit-mass system

$$k_p = (2\pi f_n)^2 \qquad (3)$$

$$k_d = 2\zeta(2\pi f_n) \qquad (4)$$

with the end-effector control $f = -k_p(x - x_d) - k_d(\dot{x})$. Thus, the compliance at the tool is directly adjusted. This can improve force control of hydraulic manipulators like excavators and rock breakers [13] without the need of direct loop closure in force measurement.

As a convention adopted in this paper, feedback controllers will be referenced henceforth by their parameters f_n and ζ as defined in (3) and (4).

The controller is implemented as shown in the upper block diagram in Fig. 8, where J is the Jacobian matrix, FK is the forward kinematics, $\boldsymbol{\tau}_{fric}$ is the nonlinear friction and the joint space dynamics projected in the Cartesian space are

$$M_x = {J^{-1}}^T M(\boldsymbol{q}) J^{-1} \qquad (5)$$

$$\boldsymbol{v}_x = {J^{-1}}^T \boldsymbol{v}(\boldsymbol{q},\dot{\boldsymbol{q}}) - M_x \dot{J}\dot{q} \qquad (6)$$

$$\boldsymbol{g}_x = {J^{-1}}^T \boldsymbol{g}(\boldsymbol{q}) \qquad (7)$$

B. Parameter Sensitivity in Excavator Compensation

Friction levels are high in hydraulically-actuated machines. Small parameter estimation errors can lead to excessive friction compensation in $\boldsymbol{\tau}_{comp}$ and consequent instability. This effect is shown by simulation in Fig. 3 by purposely increasing and decreasing viscous friction in the model used for compensation. When friction in the model is exactly the same as the friction in the plant, compensation eliminates the effective friction seen by the PD loop, yielding the ideal case. When friction in the model is lower than the friction in the plant there is residual friction in the system and the feedback loop has to account for that by generating extra closed-loop commands. The worst case occurs when friction in the model is higher than the friction in the plant, causing the compensation loop to add more control than is necessary, eventually leading to instability. In the viscous case, this is worsened by the fact that this energy is proportional to velocity, thus generating positive feedback.

C. Experimental Platform Issues

Although counter-intuitive, a hydraulic excavator arm *is not* as stiff as it looks. Compliance is a combination of several effects but is due mainly to expansion of flexible hydraulic hoses and the presence of entrapped air in the circuit [14]. Indeed, hydraulic compliance is the limiting factor for the feedback loop gain.

The excavator used in this work is a 1.5 tonne Komatsu PC05-7 (Fig. 1). Its working end is a three link arm with a mass of 110 kg driven by hydraulic cylinders that are actuated by servo-valves (for details see [15]). It can reach three meters when fully extended.

Fig. 4 shows the issue of compliance in the hydraulic circuit. The model used in this work reflects compliance as damped springs in the cylinders. The values of the

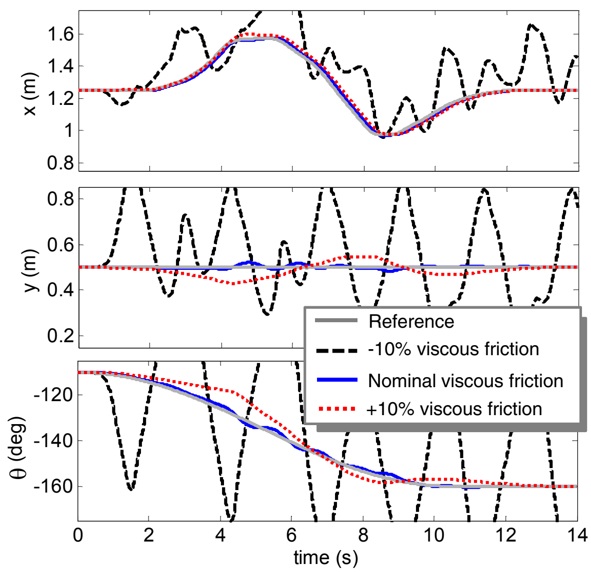


Fig. 3. The effect of friction variation on control stability shown as task-space motion of the end-effector (bucket tip). Here, the solid line represents perfect matching between compensation model and plant model response. The dotted line shows the case when friction in the plant is 10% higher than expected by the controller. There is residual friction in the system, which actually increase stability. The dashed line shows the case when friction in the plant is 10% lower than the expected value causing the inverse model to overcompensate.

boom, arm, and bucket spring natural frequencies are 2.5, 5.0, 5.0 Hz with damping ratios (ζ) 0.25, 0.70, and 0.25, respectively. These natural frequencies are in accordance with other models of excavator vibration [16], [17].

A second issue with the experimental platform concerns friction. As reported in [18], hydraulic systems have high amounts of friction due to the tight sealing required by the high pressure in the circuit (180 bar in the case of the ACFR excavator), which is worsened by the low precision of joint assemblies.

III. Friction Compensation

Friction is one of the main forces to be compensated in heavy machinery, and yet its behaviour is significantly less well understood than other dynamic effects. Friction forces also change markedly during operation. Much of the vast literature on friction modelling (for survey see [19]) builds upon the LuGre model [20] and the seminal work of Dahl [21]. Use of these models requires various adjustments ranging from parameter tuning to ad-hoc modifications, which makes generalised friction compensation very difficult to achieve.

Concerning hydraulic machinery, when the LuGre model is used as a friction compensator to control an arm with rotary motors [18][1] or a prismatic cylinder (as in [22]), the model equation is:

$$\frac{dz}{dt} = \dot{q} - \frac{\sigma_0}{g(\dot{q})} z \left|\dot{q}\right|, \quad g(\dot{q}) = \alpha_0 + \alpha_1 e^{-(\frac{\dot{q}}{\nu_0})^2} \tag{8}$$

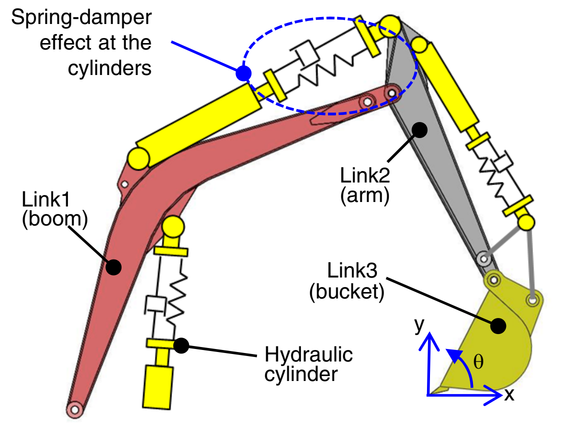
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \alpha_2 \dot{q} \tag{9}$$

where z is the internal state of friction dynamics, $\dot{q}$ is joint velocity, α_0, α_1, α_2 and ν_0 are static parameters, F is the friction torque and σ_0 and σ_1 are dynamic parameters (see [20] for a complete description of the parameters).

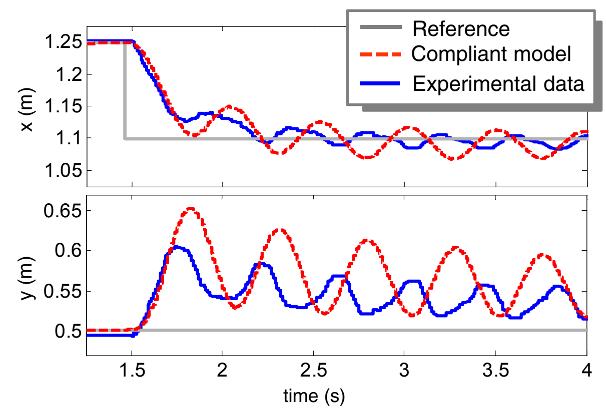
A friction estimation method for hydraulic cylinders incorporating pressure measurements was proposed in [23] with a general model described as

$$F = x_1 e^{x_2 v} + x_3(p_1 - p_2) + x_4 p_2 + x_5 v \tag{10}$$

where v is the linear velocity, p_1 and p_2 are differential pressures and $x_1 ... x_5$ are parameters to be identified. From this model, the friction characteristics of a hydraulic cylinder close to zero velocity is better described by a large viscous



a) The excavator modeled with hydraulic compliance. The tip motion is a result of a series of compliant joint motions leading to a resonant frequency of approximately 3 Hz.



b) Step response of the end-effector excited by a closed-loop controller.

Fig. 4. The excavator model designed for simulation purposes considers the hydraulic compliance as damped springs at the actuators.

[1]Estimating friction torques is only part of the problem in hydraulic machines. The other part concerns how to apply those torques at the joints. Hydraulic actuators are usually driven by direction control valves or (at best) servo valves, but not torque at joints. This requires an additional inner-loop torque control.

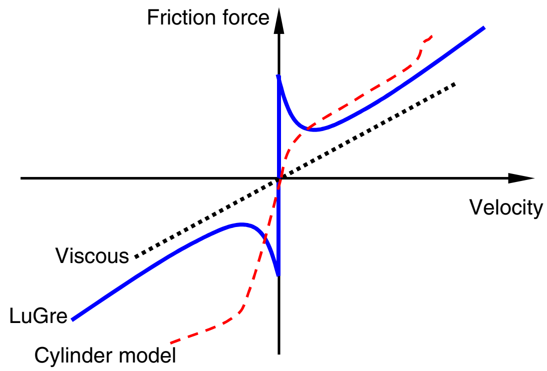


Fig. 5. An illustration of various approximate friction models.

friction: as velocity grows, it presents a smooth break-away with different magnitudes for each motion direction.

Fig. 5 shows an illustration representing the LuGre model, the cylinder friction model [23] and a simple viscous friction model. Observe that outside the region of stiction the LuGre and the cylinder models are essentially linear. This linear region may be represented by viscous friction. It is parameterised by α_2 in (9) and x_5 in (10) and is always observable whenever velocity is nonzero. It is also the only parameter that does not depend directly on the internal dynamics in (8), thus being easier to identify.

It is conjectured that for trajectories where stiction is small in temporal terms such a linear approximation yields effective compensation. An experimental exemplar of this is shown in Fig. 6 where a simplified viscous friction model overlays experimental excavation data. While point-to-point motions are dominated by regulation around stiction regimes, excavation motions (e.g. digging, dragging, and lifting) basically consist of path tracking, thus viscous friction modelling accounts for most of the motion prediction.

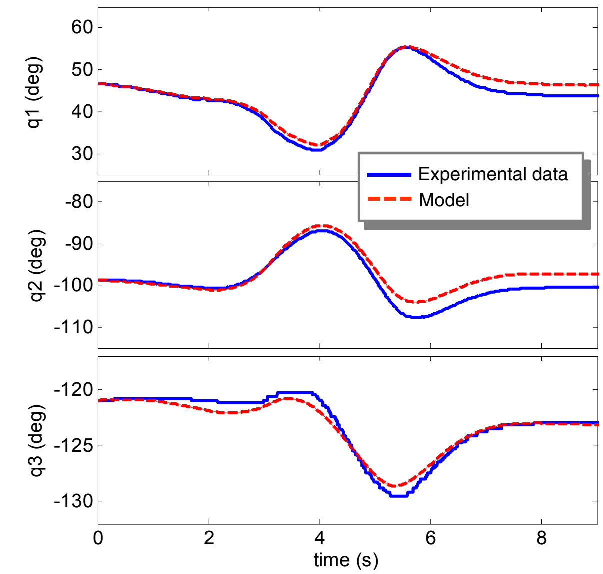


Fig. 6. Prediction using a simple viscous friction model compared to experimental data. q1, q2, q3 represent the encoder measurement of links 1, 2 and 3 respectively.

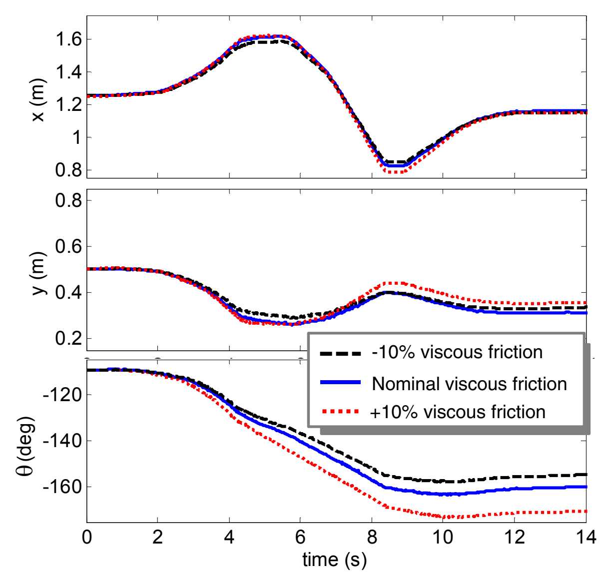


Fig. 7. End-effector motion in open-loop. Cached compensations with three different amounts of friction cause similar and stable motion patterns.

IV. Open-Loop Compensation

Dynamic compensation requires models with excellent fidelity. The results in Section III show that it is possible to predict the dominant dynamics by using a simple linear viscous friction model. So while this model gives good empirical results, its deviations (if uncorrected) will lead to positive feedback and instability.

This is in part because of errors from simplifications and parameter estimation: as described in Section II, a 10% variation in the friction parameter can destabilise a model-based controller. Notice that the difference in viscous friction has much less impact in open-loop since the damping error has a significant effect only at high velocities. At low velocities—when movements start, stop and reverse—stiction is dominant. More importantly, because open-loop compensation commands are bounded the response is also bounded.

Based on these observations, the open-loop compensation strategy shown in Fig. 8 is proposed. For a given desired motion, forward simulation is used with a previously identified simple model (in this case, viscous friction). The compensation required to generate the motion is then cached as joint commands and used in a 2-DOF controller as an open-loop command. Here the feedback loop uses the operational space controller, but any type of model-based controller could be used since compensation can be cached either as task-space commands or directly at joint level. From a control standpoint, feedback is still calculated for the end-effector position, and thus actions are calculated as if compensation was being calculated on-line. The stability

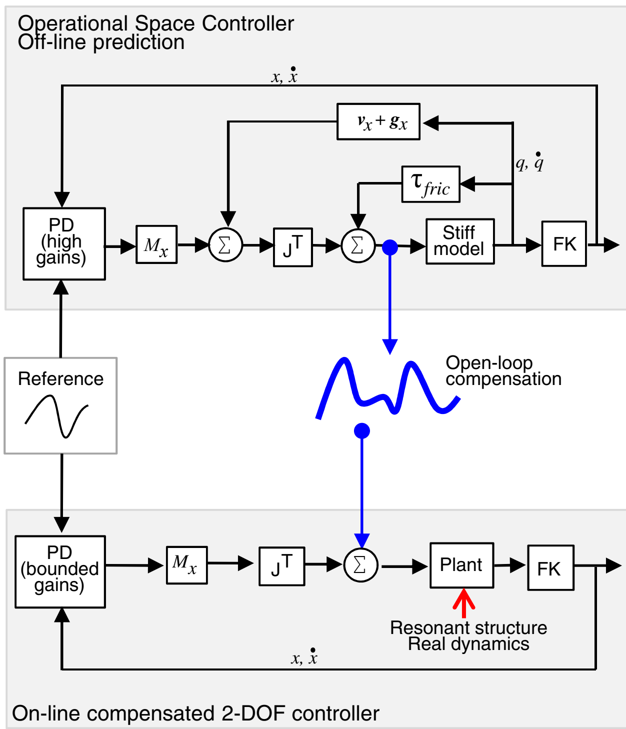


Fig. 8. The control structure uses a cached compensation from a simplified model. This signal is send as feedforward commands during experiments.

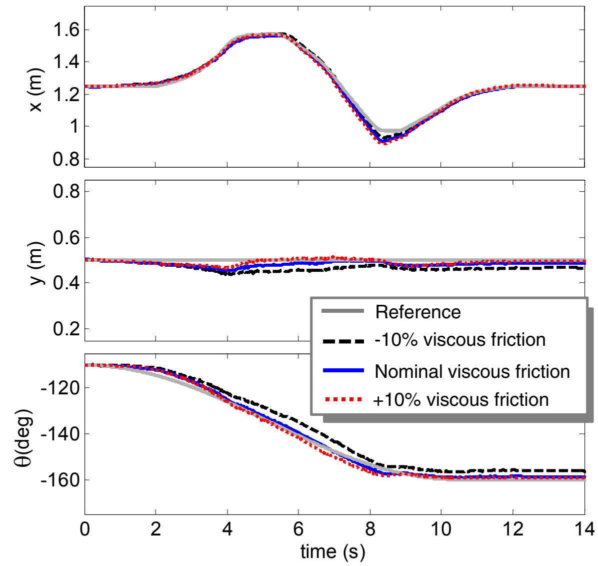


Fig. 9. End-effector motion in closed-loop. Open-loop compensations (cached with different viscous friction values) are used as feedforward commands with the operational space controller as feedback, leading to similar responses.

of the compensation comes from the open-loop property of dissipative systems. Obviously, the feedback controller must also be stable.

The idea of using prediction to generate commands in an open-loop fashion is not "new" per se. Model predictive controllers and receding horizon control (RHC) [24] implicitly act in a similar way. The parallel is evident if the one-step forward simulation in Fig. 8 is repeated on-the-fly over short time horizons. The difficulty in implementing the controller using this strategy is that friction estimation and corrective trajectories must be executed at each horizon. This may be computationally prohibitive, especially if terrain models are considered. Also important is the fact that open-loop compensation can be achieved by methods other than a single-step forward simulation (as presented here). The cached compensation could be stored and used from a library. This concept has been explored in [25] and [26] as local control laws, but could be extended to "local compensations".

Fig. 9 shows the effect of using the same set of cached compensations with different friction predictions, now in closed-loop with the 2-DOF controller. In this case, the motion is not only stable, but basically the same. Assuming compensation correctly predicts 90% of the control effort, the feedback has no difficult in compensating for the erroneous 10%. However, when on-line this could generate overcompensation and positive feedback.

V. Experimental Results

A. Tracking an Excavation Trajectory

The free-motion tracking performance of the excavator was evaluated experimentally using the reference trajectory shown in gray in Fig. 10(a) (adapted from [27]). The orientation of the end-effector starts at $-90°$ and rotates smoothly to finish with a $-160°$ posture. The solid line shows tracking when using both feedback and open-loop compensations. The dashed line, shows tracking when compensation was removed, and thus error largely increases.

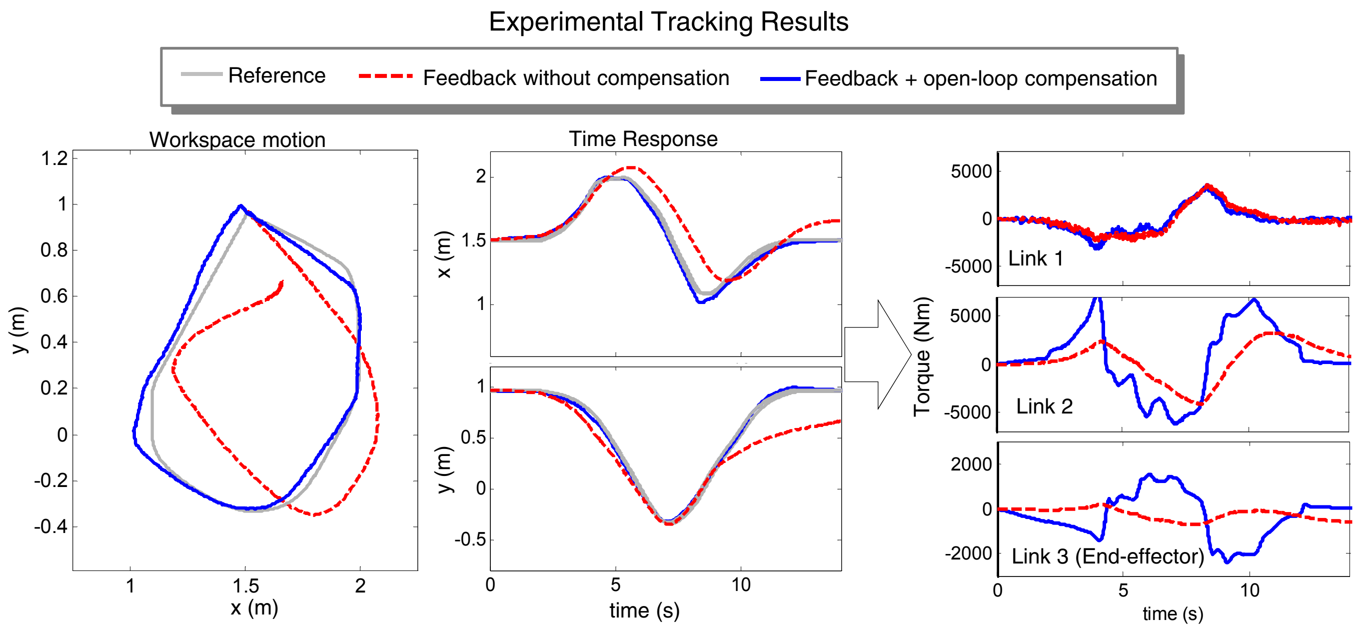
The time response (in Fig. 10(b)) indicates a huge difference in control commands. It might be intuited that since the commands from the feedback without compensation are not saturated, setting higher feedback gains would improve the results. This is not possible because of the resonant mode of the arm at approximately 3 Hz[1] (see Section II). In fact the gains used for these experiments are already at the upper bound. Compensation therefore yields increased performance without relying on high-gain feedback strategies.

B. Regulation with Step Inputs

Fig. 11 shows a 10 cm step response in free motion of the end-effector along the horizontal X-axis. The experiment was repeated in three different conditions, all of them with a feedback loop closed at 3 Hz:

1) without compensation.

[1]Note that the low gain issue due to arm compliance were not reported in previous works with excavators since control was executed in joint space. The advantage of joint space in this class of machines is that the individual loops are blind to the total motion of the arm, thus resonance of the arm is not excited "as a whole". The gains are essentially limited by the bandwidth of each individual link



a) A smooth excavation motion where the end-effector (the tip of the bucket) moves in the X-Y plane

b) Torque commands

Fig. 10. Tracking in task-space is greatly improved by the open-loop compensation. Notice that although the torque commands are not saturated when there is no compensation, higher gains could not be used due the resonance of the structure. Compensation is one effective method to improve performance without relying on high gains.

2) with open-loop compensation calculated by simulation with a 3 Hz controller.
3) with open-loop compensation calculated by simulation with a 12 Hz controller (this case using a perfect stiff model that neglects hydraulic compliance).

The case without compensation has a very slow response since all control effort is given by a reactive feedback (shown as dotted lines). In the second case (shown by the dashed lines), the rise time improves considerably but then the settling is slow and compliant due to the low gains necessitated by the resonant structure. The third case (solid line) shows a faster settling time as if the controller had higher gains despite the on-line controller tuned at 3 Hz. In fact, the compensation obtained from a 12 Hz controller in simulation over-compensates the on-line 3 Hz controller. This shows that the real performance can be improved by imposing a behaviour by open-loop commands that goes beyond what a perfectly tuned feedback controller could do (even with perfect compensation), as the latter is bandwidth-limited.

The use of aggressive inputs is only possible because structural excitations in open-loop are quickly damped by the high friction of the mechanism. If this were not the case, reshaping the trajectory to obtain smooth pre-calculated compensations becomes important.

The large steady-state error is caused by stiction, which should be compensated by a complete friction model with stiction. No integral gain was used here.

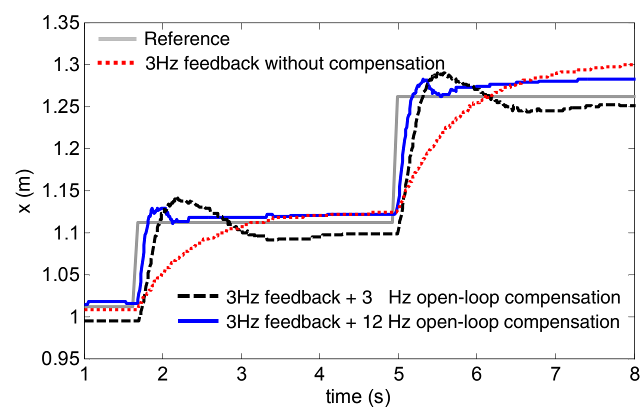


Fig. 11. A sharp step input regulated by the same feedback controller with different levels of open-loop compensation. Notice: (1) the lack of stiction modelling which is not being compensated, and (2) the measurement resolution of 5 mm.

VI. Conclusions and Future Work

This work presents a method to compensate and dynamically decouple manipulator mechanisms whose dynamics are difficult to model due to high friction levels. The compensation is obtained by using a simplified model that represents only the dominant motion regime, which is much easier than modelling the whole spectrum of possible motions. Because of the limited range and sensitivity to parametric error, this model is best suited as a feedforward open-loop command predictor. Stability is guaranteed and compensation is effective, as shown by experimental results.

Pre-calculated compensation simplifies the implementation of model-based controllers. In this work it is applied to drive a hydraulic excavator arm in (free air) task-space by using the operational space controller. Cached operation allows for high gain simulation (beyond the real resonant dynamics) and can be used to generate aggressive open-loop commands by taking advantage of the fact that excitations in open-loop are quickly damped by high mechanical friction.

In this work no terrain reactions were considered. For the presented free motion case, a simplified model based on viscous friction was used. Future work will address excavation reactions that should be added to $\boldsymbol{\tau}_{dist}$ in (1). This is expected to be achieved by using analytical models and parameter estimation as in [3], [4] that can quantify the required forces. In [28] the feasibility of a theoretical flat blade model seem limited for prediction of excavation and learning methods were proposed. This suggests the possible use of regression methods based on non-parametric or semi-parametric techniques like those presented in [11], [12] to predict reactions based on experimental data.

VII. Acknowledgments

The authors would like to thank Craig Rodgers and Pak Hung (Victor) Chan for their support with the experimental platform. This work is supported by the Rio Tinto Centre for Mine Automation and the Australian Centre for Field Robotics, funded in part by the New South Wales State Government.

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