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# Inverse Kinematics for Optimal Human-Robot Collaboration

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## Abstract

Learning directly from natural human movements provides an alternative to inefficient and sometimes infeasible kinesthetic teaching. In particular, in the field of robotic co-workers, flexible approaches to obtain collaborative trajectories becomes important, not only to avoid disruptions in the production cycle, but also because of the repetitive and abundant number of human movements that occur, for example, during an assembly task. This project aims to find such collaborative trajectories by using an inverse kinematics based approach. The solution must be optimal with respect to accuracy at which the original demonstration can be reproduced, joint limits and efficiency in human robot collaboration. Furthermore, the flexibility of the algorithm with respect to different workspaces is evaluated. The results indicate that the algorithm is capable to adapt natural human demonstrations that take place in arbitrary locations, to the embodiment of a 7-DoF robotic arm in different types of tasks.

## 1 Introduction

Developments in the fields of mechanical engineering, mechatronics and safe control enable us to consider the use of robots for a growing field of applications. In particular, the use of robotic co-workers is a challenging part of this development towards collaboration between humans and supportive machines. A main issue in the case of human-robot collaboration is how to teach robots the required abilities for a certain task. While methods such as kinesthetic teaching can be both, time consuming and sometimes even infeasible for inexperienced workers and non-backdriveable

robots, a very natural and intuitive way of teaching is learning directly from natural human demonstrations, that is, demonstrations where the human movement is not influenced or constrained in any sense by the existence of a robot learner. As human apprentices learn their part of a task by watching experienced workers, it seems sensible to let a robot learn from watching humans executing a specific task.

A requirement for the demonstration process is simplicity in the setup and in a generalizable form. In the specific case of humanoid robots, joint to joint mapping using optical markers is a widely used technique. However, multiple marker tracking in assembly tasks are prone to occlusion and the required setup for each demonstration can become a disruptive and time consuming process. An alternative to this problem is provided by recording only the hand or wrist position of the human demonstrator and subsequently using inverse kinematics to obtain the robots joint positions. Moreover, this approach provides the robot the opportunity to execute the movement in its own way (e.g. movements the favor the robot rest posture) rather than to blindly follow human motion schemes, as they are provided by joint to joint mapping.

In order to keep the learning process as general as possible and to provide solutions for arbitrary workspaces, it is additionally desirable to decouple the learning scenario from the real robot application. In this case the algorithm must address the case where human demonstrations occur in arbitrary locations and later mapped into the actual workspace of the robot.

This mapping, however, requires a search for the appropriate location of the reference frame of the original demonstration in relation to the robot's reference frame. This project proposes an algorithm to search for such a transformation of a given demonstration into the robots workspace while at the same time trying to ensure optimal human-robot collaboration during the execution of the task.

## 2 Related Work on Natural Human Demonstration

According to Argall, et al. [2], learning from demonstrations can be categorized by two main criteria: record mapping and embodiment mapping. The first regards the recording of the teacher’s execution, and the second regards how the student, i.e. the robot, executes the recorded motion with its own embodiment. The choice of the record mapping determines how the motion of the demonstrator is captured during his/her demonstration and which information is handed to the robot later. In the case of humanoid robots the question often boils down to the number of joints to track and how to extract them from the given demonstration. Several approaches such as Luo [9] use a Kinect camera and skeleton tracking in order to perform joint to joint mapping afterwards. While using the Kinect provides disadvantages in viewpoint dependent skeleton tracking, Lee, Ott and Nakamura propose a 3-D marker controlled approach for robot joint control [8]. They use a virtual spring model for each joint to let the robot follow a desired trajectory in joint space. However, one remaining problem with joint to joint mapping is that it constrains the robots solution to only imitate the human. Moreover in certain tasks, it is not even important to imitate the whole movement but to reach or track a certain end-effector position during the execution.

An alternative approach is given by recording only the human end-effector position during the demonstration. In this field of inverse kinematics (IK), the aim is to find a mapping from task to joint space. Due to redundancy, a variety of solutions and approaches exist such as robust inverse kinematics [3], style based inverse kinematics [6] and null space control [5].

The second big issue in learning from demonstrations is the embodiment mapping, also referred to as the correspondence problem [10]. It tries to resolve the parts of the demonstrator which match certain parts of the robot. Several approaches exist such as the ALICE algorithm[1] by Nehaniv, Alissandrakis and Dautenhahn. In this case a library of correspondences between the teacher and the imitator [1] is build and subsequently used to resolve the correspondence problem during demonstrations.

In this project an anthropomorphic robot arm was used and the correspondence problem was avoided by only recording the end-effector position and obtaining the joint positions through inverse kinematics. Within this approach an additional focus was set on optimal human-robot collaboration as well as on the decoupling of the demonstration location from the actual robot workspace.

## 3 A Location-free Inverse Kinematics Algorithm

To obtain optimal human-robot collaboration out of a human demonstration the algorithm is structured in three main sections. As a preprocessing step, the human trajectories when working in the robot’s workspace are captured and a Gaussian mixture model is fit into it. This mixture model is used as a surrogate for the preferred locations of the human when working with the robot. Subsequently, the end-effector movements of two humans accomplishing a collaborative task are recorded by a motion capture camera system. These demonstration can happen in any arbitrary place and need not necessarily be connected to the final robot application workspace. As the goal is to replace one of the human demonstrators by the robot, one of the trajectories is labeled as human-trajectory  $\tau_h$  and the second as robot-trajectory  $\tau_r$ . Currently, this choice is made manually.

Figure 1 illustrates this main structure and reveals how human demonstration and work-space extraction become part of the optimization to achieve a collaborative pair of trajectories. The algorithm computes a transformation of the recorded trajectories into the robots workspace  $\tau(\theta)$  using gradient descent with respect to the sufficient accuracy in the robots motion, represented by the error of the IK-solution  $e^{[k]}$ , and comfortable working conditions for the human, represented by the Gaussian mixture model of the workspace extraction. As shown in figure 1, this optimization runs until the convergence of the transformation, leading to the optimized parameters  $\theta$ .

In the following, the workspace extraction, as well as the transformation  $\theta$  and the cost function  $\phi$  of the optimization are described in more detail.

### 3.1 Encoding Preferences of Human Location

To adapt a collaborative trajectory  $\tau = (\tau_h, \tau_r)$  to the final robot workspace, as a preprocessing step, a trajectory  $\tau_{ws} = (x_1, x_2, \dots, x_n)$  of a human working in this workspace is recorded. A Gaussian mixture model  $\mathcal{N}(x_n|\mu, \sigma)$  is fitted into this trajectory to ensure that the following optimization tends to let the human stand in places that are more natural and not restrictive to him. To obtain the Gaussian mixture model, expectation maximization [4] is used by iterating till convergence, where in the E-step the posterior distributions for each mixture component and for all data points  $\alpha_{nj} = p(j|x_n)$  are computed and in the M-step the parameters of the Gaussian mixture model as well as the weighting terms  $\pi_j$  are updated by using weighted estimates.

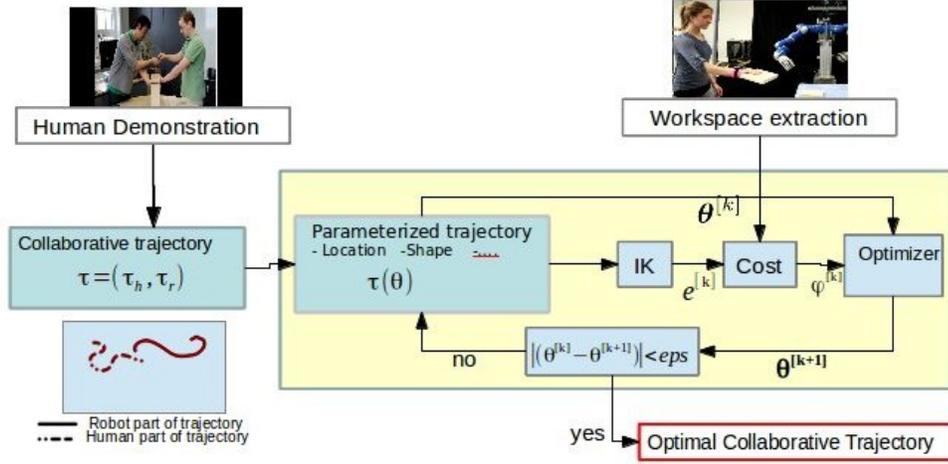


Figure 1: Main structure of the algorithm

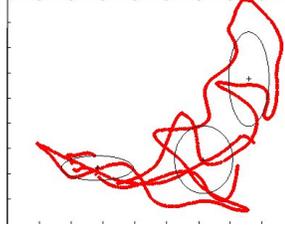


Figure 2: Gaussian mixture model from human workspace extraction

E-step:

$$\alpha_{nj} = \frac{\pi_j \mathcal{N}(x_n | \mu_j, \sigma_j)}{\sum_{i=1}^M \pi_i \mathcal{N}(x_n | \mu_i, \sigma_i)} \quad (1)$$

M-step:

$$\begin{aligned} \mu_j^{\text{new}} &= \frac{1}{N_j} \sum_{n=1}^N \alpha_{nj} x_n \quad \text{with} \quad N_j = \sum_{n=1}^N \alpha_{nj} \\ (\sigma_j^{\text{new}})^2 &= \frac{1}{N_j} \sum_{n=1}^N \alpha_{nj} (x_n - \mu_j^{\text{new}})^2 \quad \pi_j^{\text{new}} = \frac{N_j}{N} \end{aligned} \quad (2)$$

An example for a trajectory  $\tau_{ws}$  and the resulting Gaussian mixture model with three clusters is given in figure 2(a), where the covariance is shown as the ellipse.

### 3.2 Inverse Kinematics for optimal Human robot collaboration

During a collaborative task the end-effector positions of two human co-workers are captured using an optitrack system. The resulting collaborative trajectory  $\tau = (\tau_h, \tau_r)$  consists of the human trajectory and the matching counter part that should be performed

by the robot. Given this collaborative trajectory  $\tau$  a transformation into the robots workspace should ideally satisfy two main requirements. First, it should allow the robot to follow the required trajectory  $\tau_r$  with sufficient accuracy while avoiding joint limits as well as unnecessary movements. Second, it should consider the human role in the collaborative task such that the human is not forced to stand in an uncomfortable or infeasible location in the robot's workspace after the transformation. Such an uncomfortable spot can be given by obstacles such as tables or chairs as well as locations that force the human to stand too close to the robot.

The robot trajectory in task space is given by its position and orientation  $\tau_r = (\mathbf{x}, \mathbf{q})$ , where  $\mathbf{x}$  stands for the three dimensional Cartesian coordinates, and  $\mathbf{q}$  the corresponding quaternion. During the optimization, the position of this trajectory is changed using a translation in x- and y-direction and a rotation around the z axis  $\theta = (x, y, \phi)$ . Currently, we decided for not optimizing the height and roll and pitch of the demonstrated trajectories as they do not lead to changes of interest for human-robot collaboration. We assume the height at which demonstrations were executed should not be modified in relation to the orig-

inal human-human demonstrations. The mentioned requirements are incorporated in a cost function  $\phi$ , which consists of three parts. The first part  $\phi_{acc}$  ensures a sufficient accurate performance on the task, the second part  $\phi_{hum}$  pays attention to a good feasibility of the human part in the task and the last part  $\phi_{lim}$  takes care of limitations such as the robots joint limits or its workspace boundaries. The cost function  $\phi$  is given by

$$\phi = \phi_{acc} + \phi_{hum} + \phi_{lim}. \quad (3)$$

To obtain the optimal transformation of the collaborative trajectory gradient decent with random restarts is performed on this cost function until convergence of  $\theta$ . The next sections explain the single parts of this cost function more detailed.

### 3.2.1 Accuracy

In this work, an important factor in the optimization process is the accuracy that the robot can achieve while following the demonstrated trajectory. Given a trajectory  $\tau_r$  in task space the inverse kinematic of the robot are used to obtain a configuration in joint space  $\Omega$ . Whenever the robot is not capable to follow a trajectory outside his reach the inverse kinematics will give no solution, or one that deviates significantly from the desired end-effector positions in  $\tau_r$ , when put through the forward kinematics. Therefore the differences between the desired positions in  $\tau_r$  and the positions reached by the forward kinematics  $\tilde{\tau}_r$  are computed and used to assess the accuracy of the robots motion. The error between  $\tau_r$  and  $\tilde{\tau}_r$  is computed by using the sum of squared distances between the positions and the orientations between the quaternions

$$\begin{aligned} \phi_{acc} = \alpha_{acc} \sum_{n=1}^N (\mathbf{x}_n - \tilde{\mathbf{x}}_n)^2 \\ + \alpha_{orient} (\cos^{-1}(\mathbf{q}_n \cdot \tilde{\mathbf{q}}_n))^2. \end{aligned} \quad (4)$$

In this equation  $\alpha_{acc}$  denotes a factor to tune the importance of accuracy in the optimization and  $\alpha_{orient}$  provides a weighting for correct orientation compared to position accuracy. Moreover the inverse kinematics provides the possibility of incorporating a desired rest posture to ensure, for example, pleasant looking movements or energy efficient solutions.

### 3.2.2 Human factor

A second factor during the optimization of the collaborative trajectory is the resulting position for the human co-worker. It is neither desirable to obtain a trajectory where he is forced to stand at an infeasible location nor should he be moved too far from his natural working area. Therefore a human factor in the cost

function is computed considering the Gaussian mixture model, where each  $i$ -th component  $\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_j, \boldsymbol{\sigma}_j)$  is created from the human workspace extraction. This ensures that the human part of the trajectory ends up close to one of his most desired locations, extracted during previous observations. As it is assumed to be close to the human location the starting point of the human trajectory  $\mathbf{x}^*$  is used to compute the cost function term  $\phi_{hum}$  for the optimization

$$\phi_{hum} = -\alpha_{hum} \sum_{i=1}^M \pi_i \mathcal{N}(\mathbf{x}^*|\boldsymbol{\mu}_j, \boldsymbol{\sigma}_j) \quad (5)$$

To provide the possibility of tuning the importance of a comfortable or feasible working spot for the human,  $\alpha_{hum}$  is used as a weighting factor.

### 3.2.3 Robot limits

To avoid damage on the hardware as well as to ensure a smooth movement it is important to stay away from the joint limits and to avoid stretched out robot positions. Those requirements are met by the limit term  $\phi_{lim}$  in the cost function that cares for both joint limits and the distance to the workspace boundaries.

To avoid a violation of the joint limits  $\Omega_{max}$  a force similar to a spring is computed out of the distance to the joint limits  $\Delta_\Omega$ . Additionally the distance to the workspace boundary is computed by using the sum of squared distances between the points of the robot trajectory  $\tau_r$  and the center of the robots workspace  $\mathbf{x}_{center}$ .

$$\phi_{lim} = \alpha_{joints} \sum_{n=1}^N (\Omega_{max} - \Delta_\Omega)^2 + \alpha_{ws} \sum_{n=1}^N (\mathbf{x}_n - \mathbf{x}_{center})^2 \quad (6)$$

The weighting factors  $\alpha_{joints}$  and  $\alpha_{ws}$  are used to tune the importance of the limit terms with respects to accuracy and human factor.

### 3.2.4 Optimization

Given a pair of collaborative trajectories  $\tau = (\tau_h, \tau_r)$ , a parametrization is used to map these trajectories into the optimal position in the robot's workspace. This parametrization is given by translating and rotating the reference frame of the demonstration in the x-y-plane  $\theta = (x, y, \phi)$ . The algorithm aims to find the optimal transformation with respect to the defined cost function

$$\theta^* = \operatorname{argmin}_\theta \phi(\theta) \quad (7)$$

The optimal parameters are obtained using gradient descent with random restarts, to gain efficient computation time while avoiding local optima.

## 4 Evaluation in Real Human-Robot Handovers with a 7-DoF Arm

The performance of the proposed algorithm was tested in different simulated and experimental settings. In this project, a robot setup consisting of a 7-DoF KUKA lightweight arm equipped with a 5-finger hand was used as shown in 3(a).

### 4.1 Simulation

A reliable simulation environment provides the advantage of avoiding damage on hardware and also ensures efficient offline testing. The Robot Operating System (ROS) and the Unified Robot Description Format (URDF) were implemented to obtain a kinematic model of this robot. Figure 3 displays both, the robot and the URDF model used for simulation. To also obtain a better insight of the algorithm, the ROS rviz was used together with a ROS node to visualize the robots movement as well as the optimization of the given trajectories. The simulation was used to evaluate first toy examples, such as a simple line following, or basic circle movements. In particular, in the case of three dimensional trajectories, visualization of the process also provides the advantage of deeper understanding of the optimization evolution. A screenshot of the simulation environment is shown in 3 (b).

### 4.2 Experiments

To test the algorithm two main types of experiments were performed. The first was given by a simple imitation of a human movement. The second set of experiments focused on optimal human-robot collaboration.

#### 4.2.1 Imitation of human movement

For humans, imitation is a very natural way to learn a new task. Also a robotic co-worker might face sequences where he is supposed to perform a specific movement that was shown to him before by an experienced worker. Therefore in the first experiments a human demonstrator performed a specific movement, such as a circle with his arm, and the robot was supposed to repeat this movement while only using the recording of the human end-effector. This recording was obtained using the motion capture system Optitrack and tracked marker on the wrist of the demonstrator. To test the algorithms capability to transfer demonstrations from arbitrary locations to the robot's workspace, the demonstrator changed his position and orientation during the experiments.

#### 4.2.2 Optimal human-robot collaboration

To acquire human-robot collaborative trajectories it is sensible to make use of demonstrations of humans performing a collaborative task. Due to the applicability in multiple working scenarios, a simple handover was chosen. In this setup two humans performed the handover while again their end-effector positions were tracked by a motion capturing system. The demonstration happened outside the robots workspace and could be performed in any arbitrary location. The demonstration is illustrated in figure 4 and the robot was supposed to take over the part of the right human. To enable the robot to replace this human worker subsequently also a workspace extraction of the robots workspace was performed. For this extraction we considered two different workspace settings to test the capability of the algorithm to reuse a single demonstration for multiple applications. In the first setting a table was placed on the left side of the robot blocking this part of his workspace and in the second case, the table was placed on the right side.

### 4.3 Results

The first experiments revealed the algorithms ability to find a feasible trajectory for the robot to imitate a simple human movement, without a need for kinesthetic teaching. They also indicated the advantage of being independent from the exact location of the demonstration.

In the case of the collaborative task, Figure 5 illustrates the robot performing the co-worker part. The robot was capable to perform the handover while avoiding the workspace limits. Also the human was not forced to stand at a restricted or dangerous location which ensured efficient human-robot collaboration.

Figure 6 indicates that the results of the optimization could adapt to the given workspace constraints of the robot. The collaborative handover could be performed in different directions depending on the prior workspace extraction, that considered the different positions of the table. In the case of a robotic co-worker this provides advantages due to the decoupling from the demonstration and the actual application area. The experiments also revealed that the solution respects the joint limits while avoiding undesired configurations of the robot.

## 5 Conclusion and Future Work

This project introduced an algorithm to obtain optimal pairs of trajectories for human-robot collaboration out of recorded end-effector positions only. The demonstrations avoided kinesthetic teaching and

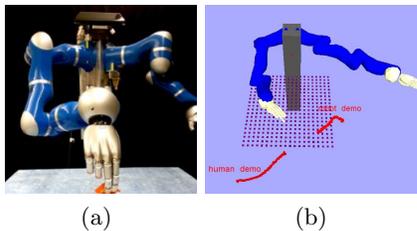


Figure 3: (a) Robot used for experiments (b) urdf model of robot used in simulation

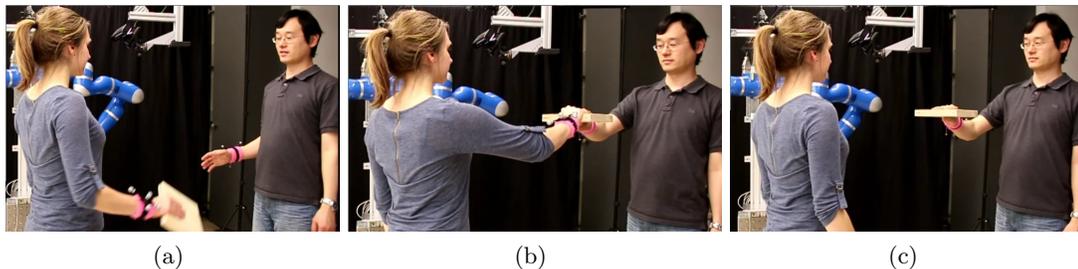


Figure 4: Demonstration of a handover task. Right human should be replaced by the robotic co-worker subsequently

could be recorded in arbitrary spaces. The algorithm was not only capable to map given trajectories into the robot workspace but also found the optimal location with respect to constraints such as joint limits or workspace requirements and preferences.

The experiments have shown basic capabilities to imitate human movement as well as to replace a human in a co-worker setup such as a simple handover task. As an advantage over other approaches such as joint to joint mapping is the reuse of the demonstrations to different workspace settings and to robots with different kinematics.

Currently, the algorithm presents two limitations. First, if the shape of the movements are fixed and during the optimization, the collaborative trajectory can be only translated or rotated, but it can not be modified. In particular, in the case of dynamic obstacle avoidance such a modification might become necessary. An possible approach for not only modifying the trajectory with respect to an obstacle, but also keeping start and end point fixed is given by trajectory based motion planning methods [7].

Second, the trajectories do not encode any form of correlation such that adaptation of the robot movement in relation to the human is not possible once the optimization is finished. Ideally, it would be possible to react in multiple ways to a certain movement. In this case an approach similar to [11] to learn not only from a single demonstration, but to use multiple demonstrations to condition over an obtained distribution is under consideration.

In both cases, knowledge provided by the robot sensors to react on dynamic changing environments must be incorporated as they might occur in a real world co-working scenario.

Future work should mainly concentrate on overcoming these shortages as well as further evaluate the algorithm quantitatively on additional experiments.

## 6 Acknowledgements

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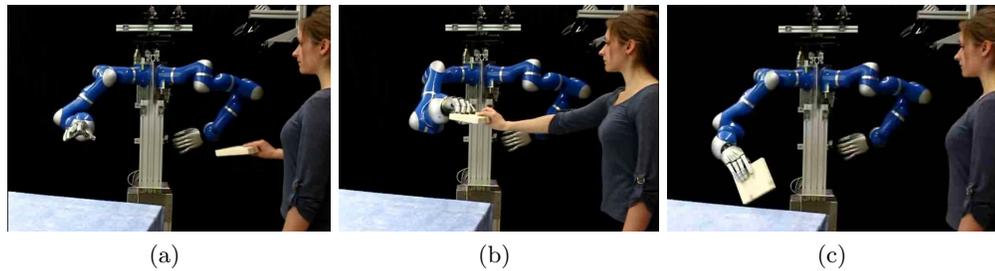


Figure 5: Robot takes his part in the collaborative task after optimization for the optimal collaborative trajectory

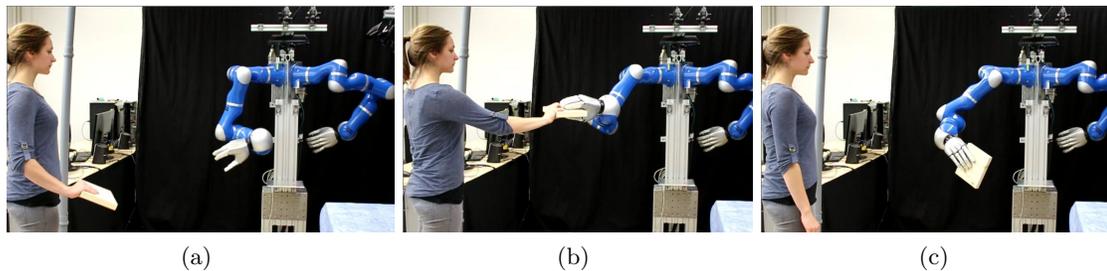


Figure 6: Using the same demonstration for a different workspace setting the algorithm is capable to adapt and still obtain a solution that ensures efficient human-robot collaboration

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